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Persistent Liquidity *

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Abstract

Using US post-war data we find evidence of cointegration between the short term interest rate, inflation, unemployment and money supply growth. Rolling trace tests add robustness by showing lack of cointegration when money or one of the other variables are omitted. Significant non-linear dynamics are found with three endogenous Markov-switching regimes, interpreted as contractions, expansions, and "unconventional" periods. We interpret the results in terms of a persistent liquidity effect with distinct dynamics over time as regimes shift across normal business cycle fluctuations and rare events.

JEL Classification: C32, E40, E52,

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1 Introduction

Alvarez and Lippi (2014) present neoclassical money demand theory in which there exists a persistent liquidity effect such that money supply growth can lower the nominal interest rate for a prolonged period. Money-caused liquidity effects on interest rates are also the focus of Lucas (1990), Alvarez, Lucas and Weber (2001), Reynard and Schabert (2010), Venkateswaran and Wright (2013) and Williamson (2012). Williamson finds that "there exists a permanent nonneutrality of money, driven by an illiquidity effect".

Using monthly data for the largest data US period available, from 1960 to 2012, and without imposing structural breaks, we find unit roots of each data series of the nominal interest rate (Federal Funds or Treasury 3-month bill), the inflation rate, the unemployment rate, and the money supply growth rate. We find novel evidence of one cointegrating vector amongst these four series. Solving for the interest rate, results show a positive above-one inflation rate effect, a strong negative unemployment effect, and a negative money supply growth effect that we interpret as a liquidity effect. With evidence of non-linearities in the dynamics (eg. Clarida et al., 2006), we find three endogenous Krolzig (1997, 1998) -type Markov switching regimes (MSIAH) that track contractions, expansions, and "unconventional" or rare event times. During both contraction and expansion regimes the past unemployment rate changes negatively explain current short run nominal interest rate changes. For the "unconventional" third regime, its probability of occurrence correlates with negative real interest rate periods making it related to rare events especially within the post 2001 period. We view the results from the perspective of evidence supporting a persistent liquidity/illiquidity effect on the long term variables that establish the cointegrating vector that in turn explains how the economy behaves in the face of continual shocks, with inclusion of the shorter dynamics across regimes characterized by normal business cycles and rare events.[1]

Unemployment and inflation have been exploited for cointegrating vectors as based on unit roots in each series in Ireland's (1999) cointegration support for the Barro-Gordon (1983) hypothesis, as well as in Shadman-Mehta (2001) support for similar cointegration but with causality going from inflation to unemployment. Our work extends this within the purview of the relation of these variables with nominal interest rates and money supply growth. Related approaches used for Taylor rule estimation take a stationary data approach with exceptions stressing unit roots such as in Siklos and Wohar (2006) and Christensen and Nielsen (2009). We follow the latter approach in order to bring out more fully the alternative results that are found without structural breaks added to the data. Our results emphasize the economic inter-relations of key monetary policy variables, with a focus on a possibly persistent liquidity effect, rather than constructing important policy forecasts such as do Komunjer and Owyang (2012).

[1] We view the shocks as arising for example from goods sector productivity, bank sector productivity, and money supply shocks, as affecting a motivating Euler equation, for the econometric methodology, that is part of the equilibrium in the economy presented in the Appendix.

Section 2 presents the data properties, Section 3 the econometric methodology and Section 4 Robustness tests. Section 5 presents a three-state Markov switching model, Section 6 interprets some results, Section 7 provides additional discussion and Section 8 concludes.

2 Data

The empirical analysis uses United States monthly data from 1960.1 to 2012.12, in terms of the change from a year ago.[2] This eliminates seasonal variation factors and is common in this literature such as Christensen and Nielsen (2009) and Siklos and Wohar (2006). We use the Federal Funds rates for the nominal interest rate $\bar{R}_t$ (while also examining the alternative Treasury 3-month bill rate; not reported); the percentage change in the CPI for the inflation rate $\bar{\pi}_t$ [$\bar{\pi}_t = (\Delta_{12} cpi_t)100$) where cpi_t is the log of the consumer price index ($\ln CPI$)]; the log of the unemployment rate u_t (series ID : UNRATE);[3] the growth rate in $M2$ [$(\Theta = \Delta_{12} \ln M2)100$] for the money supply growth (we also alternatively used $M1$; results available upon request). Figure 2 graphs each of the four variables.

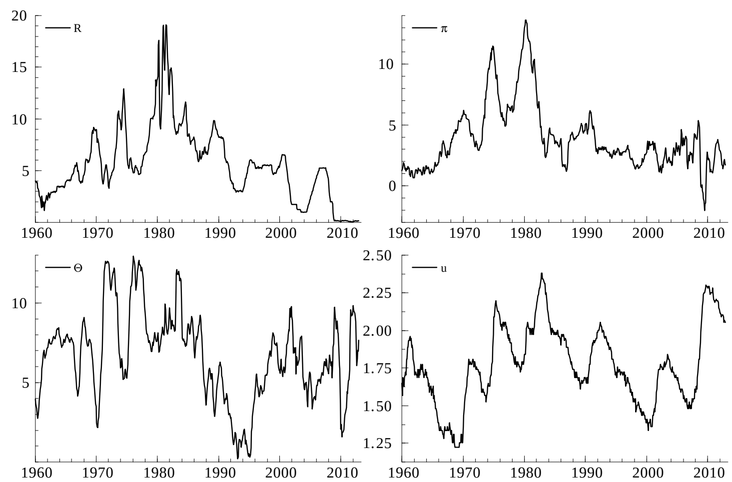

Figure 1. Federal Fund Rate, inflation rate, rate of growth of M2, log of unemployment rate.

We check for the presence of a unit root by means of the ADF test, the DF-GLS test (Elliott et al., 1996), and the PP test (Phillips and Perron, 1988), allowing for an intercept as the deterministic component. The unit root null cannot be rejected at the 5% level in all cases. KPSS stationarity tests (Kwiatkowski et al., 1992) confirm this result. Differencing the series induces stationarity (results available upon request). Adding in structural breaks is an alternative approach that we do not take. We choose

[2] We use the Federal Reserve Economic Data of the Federal Reserve Bank of St. Louis online FRED database.

[3] This log-transformation does not alter the cointegration analysis and it is consistent with the theory of the Appendix in using the rate of change in the unemployment rate as a proxy for the rate of change in our model economy representative agent's choice of leisure.

to consider that the Vietnam War period included changes in variables that we want to analyze as part of an integral fiscal and monetary policy mix and that we do not want to reduce in complexity through unexplained breaks in the data series themselves, just as we want to include the Great Recession period without artifice.[4] We are fully sympathetic to more common approaches that add structural breaks and analyze the data as stationary, leaving the breaks as exogenous. We appreciate that alternative approach but here take the data as is and offer a view based on cointegration evidence given the unit roots that are found.[5]

Year-on-year inflation data in cointegration analysis is used for example in Sauer and Sturm (2007), Christensen and Nielsen (2009), and Belke and Cui (2010). This approach can include volatility that exists in real time, during each month, that is absent from less frequent data and that we want to exploit in the sense of explaining the results within this context, and with as many data points as possible.[6]

3 Econometric Methodology

Based on the Euler equation motivation found in the Appendix (equation 6), we want to capture the expectations of four key future variables by starting with a VAR(k) system, which can be more conveniently written as a VECM($k-1$):

$$\Delta y_t = \upsilon + \Pi y_{t-1} + \sum_{j=1}^{k-1} \Upsilon_j \Delta y_{t-j} + \Sigma\varepsilon_t \ , \tag{1}$$

where $y_t = \begin{bmatrix} \overline{R}_t & \overline{\pi}_t & \Theta_t & u_t \end{bmatrix}'$, υ is the vector of intercept terms, Υ_j are matrices containing short-run information, while Π is a matrix with the long-run information of the data and $\Sigma\varepsilon_t$ is a vector of errors with $\varepsilon_t \sim i.i.d.N(0, I)$. We assume that the reduced-form shocks follow a multivariate normal distribution, $\Sigma\varepsilon_t \sim N(0, \Phi)$, where Φ denotes the variance-covariance matrix of the errors.

Using tests for lag length (details available upon request), we apply Johansen (1988, 1991) to estimate a VAR(6)[7] with a reduced rank of the long-run matrix Π equal to one (r=1).[8] Here we define $\Pi \equiv \alpha\beta'$, where α and β are (4×1) vectors. The α is a vector of "loading coefficients" describing each variable's speed of adjustment back to the long run equilibrium when significant, and the β vector contains the cointegrating coefficients of each variable.

[4] For example, in the sense of Lucas and Stokey (1983).

[5] Most unit root tests have low power but as long as the series tested are found to be I(1) and they do not have bubbles then the time series properties of the series do not need to be further constrained; new approaches that allow for such further testing are in Phillips et al. (2013). We owe this point to Pierre Siklos.

[6] We find that the month-on-month inflation rate and money supply growth rate are both I(1); results are available upon request.

[7] On the basis of AIC information criterion we choose a VAR(6) and the LM test of autocorrelation shows that there is no autocorrelation of order one in it.

[8] This finding is corroborated by looking at the roots of the companion matrix of the chosen VAR(6), which show that there are three common trends; results available upon request.

Table 1 reports the trace and the maximum eigenvalue tests, with a dummy break for the cointegrating vector used in 1991-1994, which we interpret as a proxy for a shift due to the financial deregulation ending in 1994 (see B. Friedman and Kuttner, 1992; Friedman and Schwartz, 1982, Barnett et al., 1984, Gillman and Otto, 2007).[9] We find one cointegrating relationship amongst the four variables. Table 2 reports the α and β coefficients for each variable of the cointegrating vector.

Table 1: Test for cointegrating rank

Rank	0	1	2	3
Trace test [Prob]	79.50[0.000]**	32.35[0.098]	16.12[0.172]	5.20[0.272]
Max test [Prob]	47.15[0.000]**	16.23[0.293]	10.92[0.267]	5.20[0.272]
Trace(T-nm) [Prob]	76.46[0.000]**	31.11[0.129]	15.50[0.203]	5.00[0.294]
Max(T-nm) [Prob]	45.35[0.000]**	15.61[0.339]	10.51[0.301]	5.00[0.293]

Note. The trace test and the max test are the log-likelihood ratio tests (LR), which are based on the four eigenvalues (0.072, 0.025, 0.017 and 0.008). The VAR tested for cointegration is a VAR(6) with an intercept in the cointegrating vector. The row denoted as rank reports the number of cointegrating vectors, and [prob] indicates the p-value computed from critical values by Doornik (1998). The last two rows report small sample correction.

Table 2: Cointegrated coefficients and loading coefficients

	Cointegrating coefficients β'	Loading coefficients α
$\bar{R}_t$	$\mathbf{1}$	$\boldsymbol{\alpha}_R = -\mathbf{0.012}$ $(\mathbf{0.004})$
$\bar{\pi}_t$	$-\mathbf{2.519}$ $(\mathbf{0.295})$	$\boldsymbol{\alpha}_\pi = \mathbf{0.002}$ $(\mathbf{0.003})$
Θ_t	$\mathbf{0.927}$ $(\mathbf{0.282})$	$\boldsymbol{\alpha}_\Theta = -\mathbf{0.011}$ $(\mathbf{0.003})$
u_t	$\mathbf{10.952}$ $(\mathbf{2.913})$	$\boldsymbol{\alpha}_u = -\mathbf{0.001}$ $(\mathbf{0.0002})$
$Const.$	$-\mathbf{21.475}$ $(\mathbf{5.212})$	

Note. The standard errors are presented in the round parentheses

Table 3 presents results of the cointegrating vector with two additional tested restrictions. We test the restriction $\alpha_\pi = 0$, which is not rejected implying that π is weakly exogenous. Further, the hypothesis that $\beta_\Theta = 1$ is not rejected ($\chi^2(2) = 0.44821[0.7992]$). Note that we also test to see if Θ is a relevant variable for cointegration; the LR test on $\beta_\Theta = 0$ rejects the hypothesis that it is not relevant: $\chi^2(1) = 7.301[0.0069]^{**}$. Tables 2 and 3 both show that with reference to the entire period all the variables react to the equilibrium error with the expected sign, except the inflation rate $\bar{\pi}_t$ which is weakly exogenous such as might occur with credible inflation rate targeting.

[9] The Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994 codified the end of most nationwide restrictions on bank branching, which began taking place de facto widely in the early 1990's through holding companies.

Table 3: Multivariate cointegration analysis

	Cointegrated coefficients β	Loading coefficients$a$
$\bar{R}$	$\mathbf{1}$	$\boldsymbol{\alpha}_R = \mathbf{-0.011}\ (\mathbf{0.0038})$
$\bar{\pi}_t$	$\mathbf{-2.572}\ (\mathbf{0.304})$	$\boldsymbol{\alpha}_\pi = \mathbf{0}$
Θ_t	$\mathbf{1}$	$\boldsymbol{\alpha}_\Theta = \mathbf{-0.011}\ (\mathbf{0.0029})$
u_t	$\mathbf{12.145}\ (\mathbf{3.074})$	$\boldsymbol{\alpha}_u = \mathbf{-0.001}\ (\mathbf{0.0002})$
$Const.$	$\mathbf{-23.900}\ (\mathbf{5.395})$	
	Test of weak exogeneity	LR test of restrictions:
Restriction:	$\alpha_R = 0$	$\chi^2(1) = 6.2675[0.0123]^*$
Restriction:	$\alpha_\pi = 0$	$\chi^2(1) = 0.4375[0.5083]$
Restriction:	$\alpha_\Theta = 0$	$\chi^2(1) = 9.4585[0.0021]^{**}$
Restriction:	$\alpha_u = 0$	$\chi^2(1) = 11.336[0.0008]^{**}$

Note.The standard errors are presented in the round parentheses, while the p-values are reported in the square brackets

Figure 3 graphs the computed Federal Funds rate (FFR) using the cointegration vector of Table 3 and the actual data series for the other three variables (in Blue), so as to compare the model's "predicted" variable with the actual FFR (in Red). The computed cointegrating vector fluctuates around the actual rate, with generally greater swings in amplitude than the actual. In Black, we further compute using monthly data a Taylor (1993) rule for which we substitute in deviations from trend of the Index of Industrial Production (ipi) for the output gap term (since this ipi data is monthly) and use the original 1.5 inflation rate and 0.5 output gap coefficients. Deviations from actual interest rates found in the cointegrating vector appear at times larger than for those of the Taylor rule.

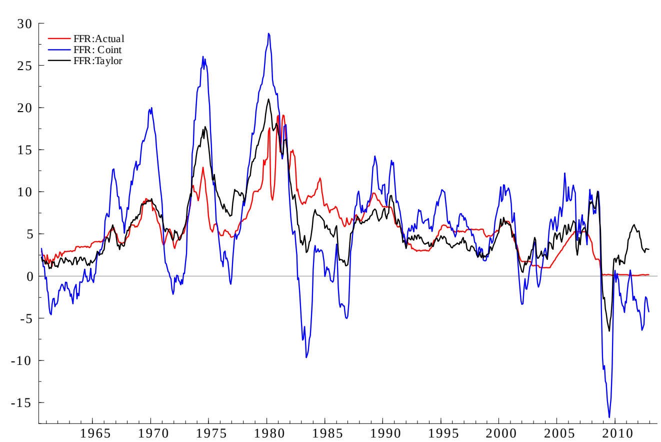

Figure 2. Actual Federal Funds rate (Red) versus Cointegration model computed rate (Blue) and a Taylor rule computed rate (Black).

4 Robustness

We have exploited for the US period from 1960-2012 that the four variables of our cointegrating vector each exhibit a unit root. This is done in view of Granger and Newbold (1974) and Phillips (1986), who show that a static regression in levels is spurious when some of the variables in the regression have unit roots. Also note that evidence of non-stationarity for US data has been reported for example by Bunzel and Enders (2005) and Siklos and Wohar (2006). During periods when the money supply growth rate tests generally as I(1) or near I(1), as well as does the inflation rate, the estimation including money growth would be expected to show better results than one without money.

For robustness we perform a rolling cointegration trace test with money and without money, plus the other three variables. We use the rolling window technique (Rangvid and Sorensen, 2002) that is based on keeping constant the window of the sub-sample and then rolling it forward through the full sample. The test statistics are calculated for a rolling 150 observation window (which corresponds to 12.5 years in our 32 year 1960-2001 sample period in this testing) by repeatedly adding one observation to the end and removing the first observation.[10] Starting with observations 1–150, we calculate the first trace test statistics; then we iteratively calculate the trace test for observations 2– 151, 3–152, 4–153 and until the end of the sample period is reached. The sequences of these statistics are scaled by their 5% critical values.

Figure 4 plots the scaled trace test statistics for the null hypothesis r = 0, against the alternative r = 1 (one cointegrating vector). A test statistic above one means that the corresponding null hypothesis can be rejected at the 5% level for the specified sub-sample period. The graph refers, respectively, to the cointegrating relation between R, π, Θ, u (the black continuous line) and between R, π, u (the dashed line). Results indicate evidence of a stable cointegrating relation for both up to the end of the 1982. Cointegration in the formulation without money disappears for most of the 1982-1999 period.[11] Estimation with money growth indicates mainly stable cointegration, with the main multi-year exception being from 1991 to 1994, plus exceptions in 1985, 1987 and 1999.

[10] Several trials with larger windows and various lags in the VAR specification have been made with similar results.

[11] A varied literature argues that such relations when estimated as static relations are candidate to be spurious regressions even if a smoothing term is included.

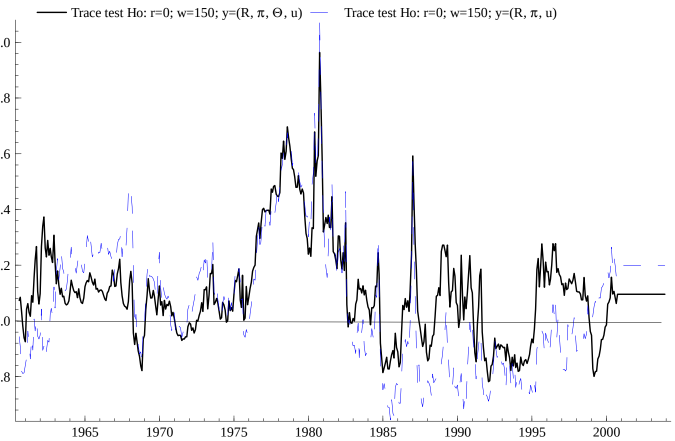

Figure 3. Rolling Trace test computed for a window equal to 150; with Euler relation, of R, π, Θ, u (the black continuous line) and without money of R, π, u (the dashed line).

The results suggest how static equation without money may be a candidate for spurious regression. Further a interest rate smoothing version of the equation may have misspecification since the Engle-Granger (1987) theorem asserts that this dynamic specification is admitted only in presence of cointegration between the involved variables. In our results, cointegration dominantly appears to exist, which may lessen the probability of misspecification bias.

Figures 4 and 5 report the rolling trace test for all possible trivariate and pairwise combinations of the four variables. Stable cointegration is indicated by values above one. Figure 4 shows that for the trivariate cases there is no clear stable cointegration in any of these combinations across the whole sample period. Sub-period cointegration episodes occur for example from 1960-1983 for R, π, and u, from 1960-1968 for R, π and Θ, and from 1962-1974 for R, Θ and u. Figure 5 shows that there exists no stable pairwise cointegration for the whole period. An example of a sub-period pairwise exception is for R and u from 1962-1978.

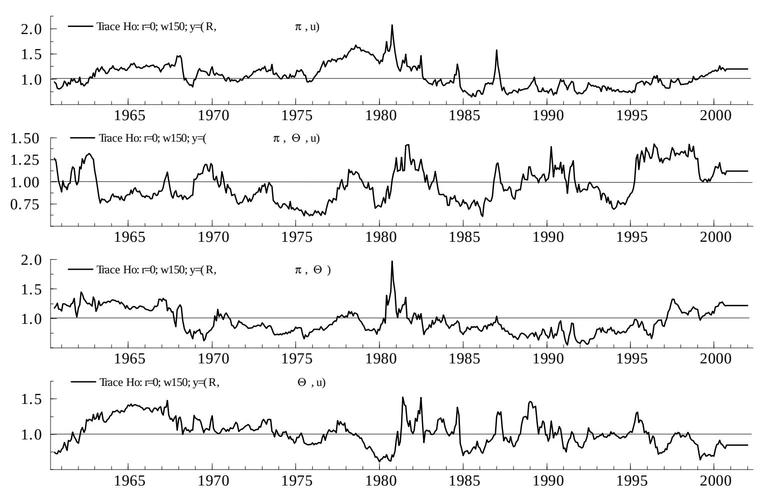

Figure 4. Rolling trace test for all trivariate combinations of 4 variables Θ, π, R, u.

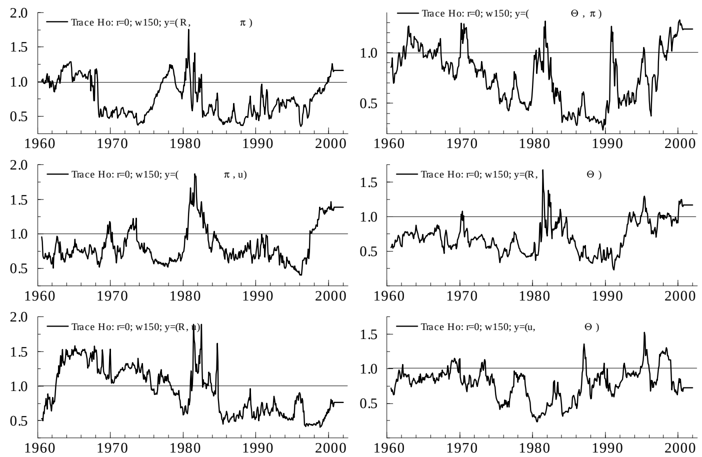

Figure 5. Rolling trace test for all pairwise combinations of 4 variables Θ, π, R, u.

5 Three State Markov-Switching VECM Analysis

We extend the analysis by including potential regime shifts in VECM dynamics because we find significant different nonlinearities in the responses of $\bar{R}_t$, $\bar{\pi}_t$, Θ_t and u_t to the equilibrium error under different regimes. As in Clarida et al. (2006), this Krolzig (1997, 1998)- type estimation approach for non-linear dynamics provides a Markov regime-switching vector error correction model (MS-VECM) that allows for state dependence in the parameters. Krolzig's procedure consist of a two-step approach: first a cointegration analysis in a standard linear model and second applying the Markov-switching methodology to account for regime shifts in the short-run parameters of the estimated VECM. This gives a multivariate linear system of non-stationary time series that is subject to regime shift, thereby capturing the non-linearities by providing alternate linear regime dynamics depending upon state.[12]

The Markov regime-switching model is based on the idea that the parameters of a VAR depend upon a stochastic, unobservable regime variable $s_t \in (1, ..., M)$. Therefore, it is possible to describe the behavior of a variable (or the behavior of a combination of variables) with a model that describes the stochastic process that determines the switch from one regime to another by means of an ergodic Markov chain defined by the following transition probabilities:

$$p_{ij} = \Pr(s_{t+j} = j \,| s_t = i), \quad \sum_{j=1}^{N} p_{ij} = 1, \quad i, j \in \{1, ..., M\}$$

The cointegrating relations are included in the MS(M)-VECM($k-1$) as exogenous

[12] The MS-VAR model by Krolzig (1997) is a multivariate generalisation of Hamilton (1989) to non-stationary cointegrated VAR systems. For this analysis it can be assumed that the error term is not normally distributed; Johansen (1991, p. 1566) shows that the assumption of Gaussian distribution is not relevant for the results of the asymptotic analysis. Saikkonen (1992) and Saikkonen and Luukkonen (1997).show that most of the asymptotic results of Johansen (1988 and 1991) for estimated cointegration relations remain valid and can be extended to include the data generated by an infinite non-Gaussian VAR.

variables, which are assumed to remain constant, where k denotes the number of lags and M the number of regimes.[13] There are many types of MS-VAR models and in this framework the model selection is more complex than in a linear model. We have to decide the maximum lag, which parameters are allowed to vary and how many regimes are to be estimated. The letters following MS stand for the respective parameters varying, specifically: I for the intercept, A for the short-run coefficients, and H for the covariance matrix. The Markov-switching MSIAH-VECM that generalizes the system (1) is

$$\Delta y_t = \upsilon(s_t) + \alpha(s_t)\beta' y_{t-1} + \sum_{j=1}^{k-1} \Upsilon_j(s_t)\Delta y_{t-j} + \Sigma(s_t)\varepsilon_t, \qquad (t = 1, .., T) \tag{2}$$

where[14] $\Sigma(s_t)\varepsilon_t \sim N(0, \Phi(s_t))$, $\Phi(s_t) = \Sigma(s_t)\Sigma'(s_t)$, $s = 1, .., z$ and the parameters $\upsilon(s_t)$, $\alpha(s_t)$, $\Upsilon_j(s_t)$, and $\Phi(s_t)$ describe the dependence on a finite number of regimes s_t. Hansen and Johansen (1998) have shown that shifts in $\upsilon(s_t)$ are decomposed into shifts in equilibrium mean and in the short-run drifts of the system.

We investigate the presence of nonlinearities by allowing regime shifts in the unrestricted intercept (I), in the adjustment coefficients (A), and in the variance-covariance matrix (H), MSIAH-VECM (also known as MSIAH-VARX, where X means that in specification (2) the equilibrium relation obtained in the first step ($\beta' y_{t-1}$) is exogenous). The model captures shifts in the mean of the equilibrium error along with shifts in the drift and in the variance-covariance matrix of the innovations. At the same time we relax the assumption of linear adjustment towards the equilibrium, letting the vector of adjustment coefficients $\alpha(s_t)$ and the matrices of the autoregressive part also be regime-dependent.

We choose the number of regimes and the model in relation to the possible combination of changing parameters, amongst the MSIAH, MSAH, MSIH and MSH alternatives, following Krolzig (1997), Sarno and Valente (2000) and Valente (2003). As a first step, within a given regime (M) and a given MS specification, we choose the best model in terms of maximum lag using the Information Criteria (IC). We then compare the various MS specifications, for each combination of changing parameters (MSIAH, MSAH, MSIH, MSH), choosing the model that dominates in terms of the IC and LR (log-likelihood ratio) tests. The model selection procedure is repeated for different regimes and the chosen models with different regimes are compared and selected with the IC and LR tests.[15] Results find that based on the IC, it is difficult to choose between the models MSAH(3)-VECM(1) and MSIAH(2)-VECM(1); there is negligible difference in terms of the dating of the regimes. Based further upon the LR

[13] In this contest the usual estimation method of parameters is maximum likelihood and, given that the state variable s_t is unobservable, Hamilton (1989) suggests using a maximum likelihood Estimation Maximization (EM) algorithm that we use; see also Krolzig (1998).

[14] Model (2) is indicated as MSIAH(M)-VECM($k-1$) and could be considered the more general model in terms of changing coefficients.

[15] For IC and LR testing see on Krolzig (1997), Sarno et al. (2004), Hansen (1992, 1996) and Garcia (1998).

test, we then choose the more general MSIAH(3)-VECM(1). Results found for the less statistically preferred two-state Markov model are available upon request.

Table 4 reports the results of the three-state Markov-switching VECM of the MSIAH(3)-VECM(1) form with statistical significance of coefficients in bold. All the tests support non-linearity (LR linearity test: 1327.2753, $\chi^2(68) = [0.0000]^{**}$, $\chi^2(74) = [0.0000]^{**}$). Moreover, the Davies (1987) upper bound test does not reject the non-linear model: $DAVIES = [0.0000]^{**}$.

Table 4: Estimated coefficients in the non linear VECM(1)

Regime 1	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.v$	**0.758**	0.150	0.021	**0.041**
$\Delta\overline{R}_{t-1}$	**0.319**	0.059	**−0.119**	−0.001
$\Delta\overline{\pi}_{t-1}$	0.033	0.179	−0.194	−0.004
$\Delta\Theta_{t-1}$	**0.741**	**−0.269**	**0.237**	0.001
Δu_{t-1}	**−12.01**	−1.302	−0.471	0.161
$\beta' y_{t-1}$	**−0.029**	−0.008	−0.001	**−0.001**
SE (Reg.1)	1.037	0.397	0.407	0.032
Regime 2	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	0.001	−0.112	**0.221**	**0.026**
$\Delta\overline{R}_{t-1}$	**0.481**	0.106	**−0.219**	**−0.016**
$\Delta\overline{\pi}_{t-1}$	**0.103**	**0.314**	**−0.190**	0.004
$\Delta\Theta_{t-1}$	0.042	−0.021	0.575	0.004
Δu_{t-1}	**−1.663**	0.032	**−0.272**	**−0.217**
$\beta' y_{t-1}$	−0.0002	0.005	**−0.009**	**−0.001**
SE (Reg.2)	0.205	0.252	0.252	0.026
Regime 3	$\Delta\overline{R}_t$	$\Delta\overline{\pi}_t$	$\Delta\Theta_t$	Δu_t
$Const.$	**0.094**	**−0.656**	**0.729**	−0.015
$\Delta\overline{R}_{t-1}$	**0.660**	**0.946**	**−0.865**	−0.025
$\Delta\overline{\pi}_{t-1}$	−0.013	**0.345**	**−0.315**	−0.006
$\Delta\Theta_{t-1}$	0.009	0.005	**0.308**	0.006
Δu_{t-1}	0.127	−1.621	−1.140	0.195
$\beta' y_{t-1}$	**−0.003**	**0.026**	**−0.029**	0.001
SE (Reg.3)	0.051	0.458	0.625	0.021

Note. Bold characters mean rejection of the null hypothesis of zero coefficients at the 95% confidence level or higher.

Table 4 shows the distinct set of the estimated parameters of the VECM in each regime, endogenously separated by Markov-switching methodology. The three distinct regimes differ with respect to the coefficients of adjustment to the equilibrium error, to the variance-covariance matrix of the innovations and to the dynamic reaction to each of the variables. Figure 6 shows the conditional (smoothed) probabilities for each of the three regimes obtained from MSIAH(3)-VECM(1), while Figures 7, 8 and 9 compare these regimes probabilities to NBER contractions (Regime 1), NBER expansions (Regime 2) and negative real interest rates (Regime 3).

Regime 1 (contractions) exhibits the highest interest rate volatility (SE= 1.037, see Table 4), a statistically significant adjustment of the interest rate and the unemployment rate to the equilibrium error of $\beta' y_{t-1}$ (with coefficients of -0.029 and $-\mathbf{0.001}$ respectively in Table 4), and an absence of adjustment of money supply growth and the inflation rate to the equilibrium error, making these two weakly exogenous. The dating of Regime 1 probabilities is broadly consistent with the findings of Sims and Zha (2006), Francis and Owyang (2005) and with NBER recessions (Figure 7). This regime captures most post-1960 recessions, except 1991, and adds one short period around 1985.

Regime 2 ("expansions") is characterized by moderate volatility of all of the variables (see the SE values for the regimes in Table 4) and tends to coincide with NBER expansions (see Figure 8). The interest rate and inflation rate do not adjust to the equilibrium error with significance, thereby indicating at least weak exogeneity (the coefficients -0.0002 and 0.005 in Table 4), while money growth and the unemployment rate do significantly adjust to the equilibrium error (with coefficients respectively of -0.001 and -0.009 in Table 4). Changes in the inflation rate however depends significantly only on changes in its past period value (with a coefficient of 0.314) making it "strongly exogenous", perhaps a result of central bank credibility in inflation targeting.

Regime 3, as shown in Figure 9, prevalently captures the more recent periods, from 2004 to 2012. This is a regime where a negative real interest rate coincides with its occurrence in 1971, and after 2002. It misses the 1980 negative real interest rate by a couple of years.

Unique features of Regime 3 are that: the interest rate, the money supply growth and the inflation rate all significantly adjust to the equilibrium error (with coefficients respectively of -0.003, -0.029 and 0.026 in Table 4); the inflation rate is not weakly exogenous; the unemployment rate is strongly exogenous with no adjustment to any of the variables; and the nominal interest rate change depends only past nominal interest rate changes and the error adjustment. This regime also exhibits the lowest volatility in the interest rate (SE= 0.051 in Table 4) and the highest volatility of both the money supply growth (SE= 0.625 in Table 4) and the inflation rate (SE= 0.458 in Table 4). These findings appear to support themes found in Dotsey and King (1986).

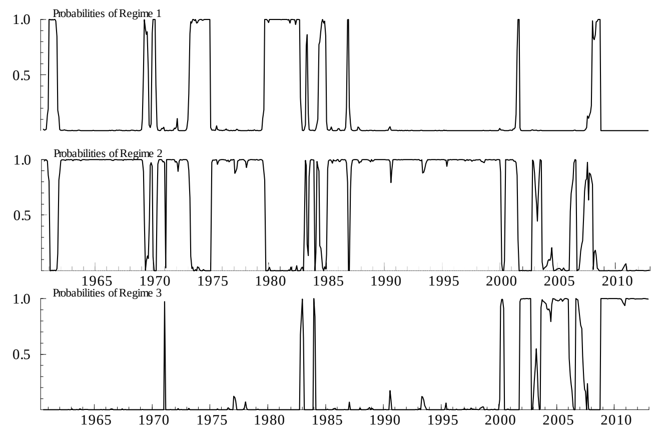

Figure 6: Conditional (smoothed) probabilities of regimes.

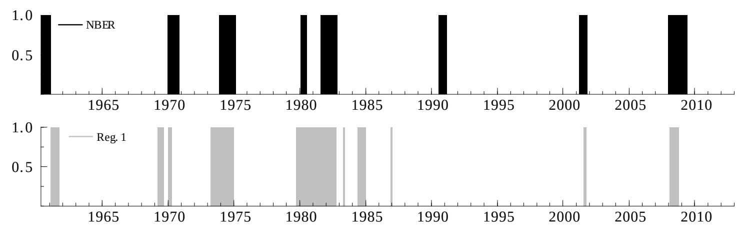

Figure 7: NBER recession dates (shadowed black areas) and Regime 1.

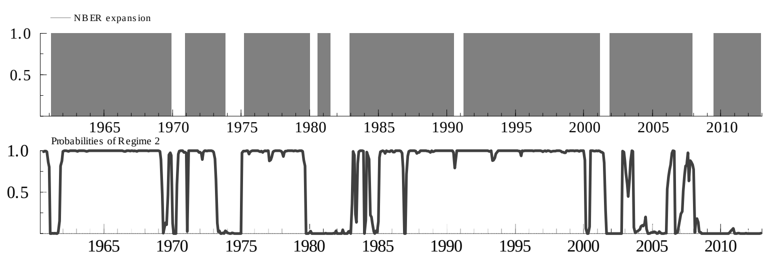

Figure 8: NBER expansions dates (shadowed grey areas) and Regime 2.

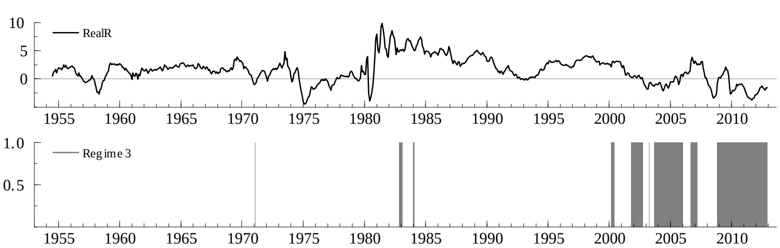

Figure 9: Real Interest Rates and Regime 3.

Table 5 reports the estimated transition matrix and the regime properties. Table 5 shows that: a) there is a lower probability to go from a recession to an expansion (0.03) than vice versa (0.08); b) there is a higher probability to remain in expansions (0.96) than to remain in recessions (0.89); c) while in expansion there is an equal probability of going to contraction as going to Regime 3 (0.08); d) when in Regime 3, there is a slightly higher probability to go to expansion (0.03) than to recession (0.02).

Table 5: Transition probabilities and Regime properties

Transition probabilities	p_{1i}	p_{2i}	p_{3i}
Regime 1	0.89	0.03	0.0002
Regime 2	0.08	0.96	0.08
Regime 3	0.03	0.02	0.92
Regime properties	*nObs*	*Prob*	*Duration*
Regime 1	103.6	0.161	9.35
Regime 2	413.4	0.648	22.94
Regime 3	112.0	0.191	11.94

Viewing monetary policy in terms of state-dependent money supply growth determinants, monetary policy may appear to be more active during expansion. For example, the past unemployment rate change negatively affects current money supply growth change only in Regime 2 (expansion); this means a falling unemployment rate causes a rising money supply growth which is possibly an active pro-cyclic policy during expansion. However the past inflation rate change negatively affects current money supply growth in both Regime 2 (expansion) and Regime 3 but not Regime 1 (contraction); this means a rising pro-cyclic inflation rate is met by a subsequent falling money supply growth rate which is more of a traditional active countercyclic policy during expansion.

6 Liquidity Effect Interpretation

Finding different non-linear dynamics that correspond to the business cycle and to negative ex post real interest rates suggests that there may be systemic differences in monetary policy that depend on the phases of the business cycles, while including rare events departures from this state-dependent system such as for financial panics (eg. post-2001 flooding of capital markets after terrorist attacks and subsequent partial fixing of nominal interest rates from 2001 to 2004, and the 2008 investment bank panic and subsequent partial fixing of the nominal interest rates from 2008 to 2015). This may give rise to periodic times when inflation rates and money supply growth rates differ from what is a priori expected. And this can cause what we know of as liquidity of illiquidity effects whereby the real and nominal interest rates can be moved away from what would exist in absence of policy responses to such events (so that the real interest rate is not at its "natural" rate).

Figure 6 graphs the actual real M2 money supply growth rate in the dashed line and the error term of the cointegrating vector in the solid line. Comparing the error term and the second dashed line gives a high correlation of 0.80.

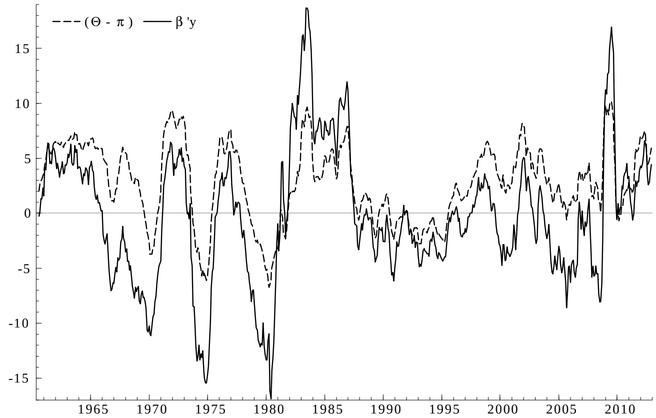

Figure 10. Growth rates of real balances (M2) and the Residual of the Cointegrating Vector.

Even though the money supply growth rate and inflation rate are part of the cointegrating vector, the difference in the two can help explain an authentic error term of what we think of as the "long run" cointegrating vector. This so-called long run of cointegrating variables is actually the persistent dynamic interaction that occurs over time as the economy experiences a continual series of shocks. Therefore it is not surprising that deviation from this long run dynamic experience coincides with a heritage of discourse linking liquidity and illiquidity effects on interest rates to deviations between the actual money supply and price level growth rates: perhaps upholding a finding of what is in some quarters constitutes a consensus view.

For example, in the lead up to the peak inflation of the early 1980s, Figure 6 shows explicitly that money supply growth was less than the inflation rate, while the error of the cointegrating vector was negative. The first effect implies a decrease in real money demand as is consistent with a rising nominal interest rate. But the second allows the possible interpretation that the expected inflation was lower than the actual, causing a lower ex post real interest rate in a liquidity type fashion. The negative error term implies that the nominal interest rose by less than was expected by the long run dynamics of the cointegrating vector. Ex post this makes the real interest rate lower as in a liquidity effect that occurred during this inflation acceleration period.

The post 1980 fall in the nominal interest rate similarly has a flavor of being less pronounced than expected by the cointegrating relation. Real money rose as it should with a falling nominal interest rate, but given the high positive error term in Figure 6, this falling real money coincided with what can be interpreted as a higher ex post real interest rate that is characteristic of a type of illiquidity effect, and consistent with an unexpected deceleration of the inflation rate (despite a falling inflation rate target - down to zero by 1988 - being enacted into law by the Humphrey-Hawkins 1978 amendment to the 1946 Employment Act). Increases in real money appear to coincide with unexpected decreases in the inflation rate. Within the cointegrating relation, a prolonged illiquidity effect of the money supply growth rate decrease resulted such that it may be viewed as resulting from an inflation rate that was lower than was expected.

The correlation of the $\Theta_t - \pi_t$ and the cointegrating vector residual echoes a Benati (2009) quantity-theoretic stylistic "fact" that inflation and money growth move together in the long run, as seen by smoothed highly correlated comovements. The qualification added by the results here are that there are persistent deviations from this comovement that affect interest rates in what might be viewed as a liquidity effect fashion.[16]

Another perspective is to view the equilibrium error in terms of Fed Chairmen. Figure 6 shows that the volatility of the error is perhaps lowest during the Greenspan tenure, consistent with Sims and Zha (2006). Perhaps this was an era of little liquidity effects on the real interest rate.

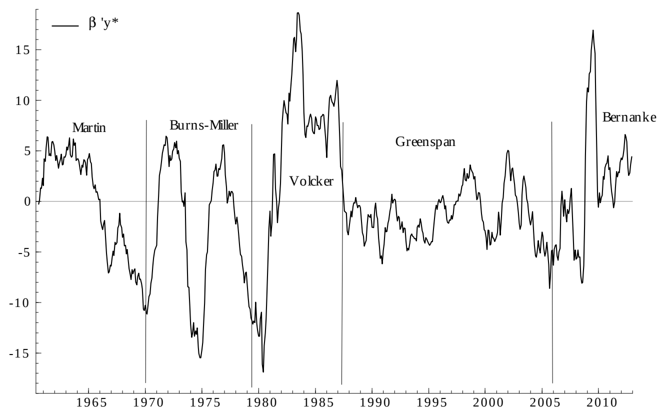

Figure 11. Equilibrium error $\beta' y_t^* = \bar{R}_t - 1.6\bar{\pi}_t + (\Theta_t - \bar{\pi}_t) + 12.2u_t - 23.9$ and US Federal Bank Chairmen's tenures.

7 Discussion

The use of M2 for the money supply was chosen for the baseline model above as it allows for cointegration with a dummy break used only in 1991-1994. Using M1 gives three similar Markov regimes and cointegration results, but requires two additional dummy breaks in the early 1980s, also coinciding with early financial deregulation legislation (results available upon request). Similarly, we check robustness by using the 3-month Treasury bill rate instead of the Federal Funds rate and find similar cointegration and regime results (available upon request), except with a preferred MSIH VECM instead of a MSIAH VECM. [17]

Interpretation of our third regime generally is presented as the unconventional period, or negative real interest rate period, but it also can be related to an "ambiguity" setting using the fixed interest rate aspect of this regime. In the recent ambiguity literature Nimark (2014) explains how the lack of a signal can induce greater uncertainty by causing the probability of the lower probability event to become assessed with a

[16] See Alvarez and Lippi (2014) for a segmented markets monetary model with possible prolonged liquidity effects. Our model in the Appendix also a form of segmentation in that "access" to credit service must be produced by an intermediary technology.

[17] We thank Charles Nolan for this point.

higher probability of occurring. We could interpret the fixing of the nominal interest rate during our Regime 3 as causing a key signal to be lost, such that it in turn raises the probability of the relatively low probability Regime 3 occurring (see Gillman et al., 2015 for a lost decade interpretation of Regime 3). And unlike both expansion and contraction regimes, our Regime 3 shows drift in the nominal interest rate change and a lack of any variable significantly explaining the unemployment rate. These facets may add to an interpretation of this regime in terms of heightened ambiguity.

8 Conclusion

The paper presents evidence of state dependent monetary policy through a cointegrated relationship between the nominal interest rate, inflation, the unemployment rate and money growth, for the US 1960-2012 period. The cointegrating equilibrium relationship is characterized by a stable liquidity effect from money supply growth as well as a greater-than-one coefficient for inflation. This liquidity effect is interpreted in terms of coinciding deviations between the money supply growth rate and the inflation rate.

Using the interpretation of the cointegrating vector residual , we show how the results can be viewed both in terms of a stable money demand along the balanced growth equilibrium and from the perspective of the Taylor rule literature, albeit without any explicit reaction function connotations. Besides the persistent liquidity effect, we find cointegration evidence in support of unemployment and inflation as key factors in the nominal interest rate determination. This can be viewed as consistent with a dual monetary policy objective in targeting both inflation and unemployment. This is qualified by a business cycle and rare event type of state dependence of the short run dynamics for each Markov switching regime.

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A Representative Agent Exchange Economy

From Benk et al. (2008, 2010), we get the following Euler equation:

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\sigma} x_{t+1}^{\psi(1-\sigma)}}{c_t^{-\sigma} x_t^{\psi(1-\sigma)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\}, \tag{3}$$

where $\tilde{R}_t$ represents one plus a ‘weighted average cost of exchange’, with weights a and $1-a$, as follows:[18]

$$\tilde{R}_t \equiv 1 + a_t \bar{R}_t + (1-a_t)\left(\gamma \bar{R}_t\right);$$

and where $\overline{R}_t$ is the net nominal interest rate, c_t is the quantity of consumption goods, x_t is leisure time, ψ is leisure preference, σ is the utililty function elasticity of substitution, and $a_t \equiv \frac{m_t}{c_t}$, which is the fraction of real consumption purchases made with real money m_t.

Log-linearization of equation (3) implies that

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\sigma E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega\psi\left(1-\sigma\right) E_t \overline{g}_{x,t+1} \\ &+ \left(\Omega - 1\right)\overline{R}\frac{a}{1-a} E_t \overline{g}_{a,t+1} - \left(\Omega-1\right) E_t\left(\overline{R}_{t+1} - \overline{R}\right); \end{aligned} \tag{4}$$

$$\Omega \equiv 1 + \frac{(1-\gamma)(1-a)}{\left(1+\overline{R}\right)\left[\gamma + a(1-\gamma)\right]} \geq 1; \tag{5}$$

$$a \equiv \frac{m}{c} = 1 - A_Q \left(\frac{\overline{R}\gamma A_Q}{w}\right)^{\frac{\gamma}{1-\gamma}} \leq 1;$$

[18] See Davies et al. (2012) for a detailed derivation; note that this is closely related to Bansil and Coleman's (1996) Euler equation.

with $a = m/c$ the BGP solution for normalized money demand, $\bar{R}$ the BGP solution for the net nominal interest rate, and γ the coefficient of labor in the production of credit q_t, where $\frac{q_t}{c_t} = 1 - a_t$. Since $\Omega \geq 1$ (=1 only if $\overline{R} = 0$ at the Friedman, 1969, optimum), the forward-looking interest rate term enters the equation, along with an inverse velocity growth term $\overline{g}_{a,t+1}$. These extra terms drop out for $a = 1$, at $R = 0$, as the equation reduces back to the form found in the simple CIA economy in which only cash is used ($a = 1$). One clear advantage of this extension for substantiating the model through empirical work is that the coefficient on the inflation term Ω is above one (for $\overline{R} > 0$) as is found also in the Taylor literature. [19]

A way to re-write the Euler equation with money supply in this case is again to combine it with the CIA constraint. This cancels out the consumption growth term, modifies the inverse velocity growth term, and adds the money supply growth term, resulting instead in a modified log-linearized equilibrium condition of

$$\begin{aligned} \overline{R}_t - \overline{R} \quad &= \quad \Omega \left(1 - \sigma\right) E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega \sigma E_t \left(\bar{\Theta}_{t+1} - \bar{\Theta}\right) - \Omega \psi \left(1 - \sigma\right) E_t \overline{g}_{x,t+1} \\ &+ \left[\left(\Omega - 1\right) \overline{R} \left(\frac{a}{1 - a}\right) - \Omega \sigma\right] E_t \overline{g}_{a,t+1} - \left(\Omega - 1\right) E_t \left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{6}$$

For motivating the econometric model, the growth in leisure is proxied by the percentage change in the unemployment rate: the possibility of rare events, plus different dynamics over business cycle recessions and expansions, is allowed for by markov switching in the probabilities of the shock variance-covariance matrix of the economies shocks. This can make the money supply growth rate and inflation rate for example follow a stochastic trend. The consumption velocity of money, which is the inverse of a, is found to be insignificant in the econometric testing and so it is not included in the reported results. Lags are introduced in the econometric model as reflective of the agent's attempting forecasts of the variables. A dummy break occurs in the econometric results which we interpret as a result of the financial deregulation of the period, with the US 1994 Banking Act, that was not picked up within the time series.

[19] See Alvarez et al. (2001) for a related approach within a segmented market economy with exogenous velocity.