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Abstract

The paper uses a monetary economy to derive a 'Taylor rule' along the dynamic path and within the business cycle frequency of simulated data, a Fisher equation within the low frequency of simulated data, and predictions of Lucas-like policy changes that shift balanced growth path equilibria and expectations. The inflation coefficient is always greater than one when the velocity of money exceeds one, thus exhibiting robust Taylor principle behavior in a monetary economy. Successful estimates of the magnitude of the coefficient on inflation and the rest of the interest rate equation are presented using Monte Carlo simulated data for both business cycle and medium term frequencies. Policy analysis shows the biases in interest rate predictions as depending on whether changes in structural parameters and expectations about variables are correctly included.

JEL Classification: E13, E31, E43, E52

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1 Introduction

After the Federal Reserve System took over buying mortgage loans from the US Treasury in 2009 and thereby turned them into "off-budget" purchases, calls have arisen to impose Congressional "rule", or law, that dictates the Fed's monetary policy.[1] The most well-known monetary policy rule is the Taylor (1993) interest rate rule. Such interest rate rule variations now dominate the older literature on money supply rules.

This paper provides a neoclassical version of what is observationally equivalent to a generalized Taylor rule. We test the theoretical equilibrium condition involving the nominal interest rate using simulated data from the model. We find that omitting variables causes model misspecification that can imply spuriously an inflation coefficient less than one. [2]

The paper next extends the horizon from the business cycle so as to include the low, or "Medium Term", frequency, as in Comin and Gertler (2006). In the medium term, the inflation coefficient falls down to one and a version of the Fisher equation of interest rates emerges. This shows a general equilibrium derivation of an interest rate rule in the dynamics of the business cycle frequency bands and the Fisher equation in the "long run" when the window is extended to include low frequency bands. [3]

Applying the rational expectations version of the interest rate solution to policy analysis, we show how an inaccurately predicted interest rate can result with a naive case of the "rule". The main example is an increase in the money supply growth rate with perfect foresight. Accurate predictions of the interest rate result by using the "rational expectations" solution to the first-order differential interest rate equation. Interest rate predictions for the effects of bank crises are also possible in this framework by allowing for a bank productivity decrease.

Section 2 describes the economy, as in Benk et al. (2008, 2010). Section 3 derives the model's 'Taylor condition' and Section 4 provides the baseline calibration. Section 5 describes the econometric methodology applied to model-simulated data and presents the estimation results. Section 6 derives theoretical special cases of the more general (Section 2) model to show how alternative Taylor conditions can be derived. Section 7 presents the policy evaluation, Section 8 the discussion and Section 9 the conclusions.

2 Stochastic Endogenous Growth with Banking

First we derive the Euler condition of the Benk et al. (2008, 2010) economy and log-linearize it to formalize how it corresponds to Taylor's (1998) more informally motivated derivation of his "rule" equation. The quantity theortic demand for money and its velocity plays the key role in producing an equilibrium version of the so-called "Taylor principle" by which the coefficient on the inflation rate is greater than one in the log-linearized asset pricing condition. We show that an exact form as in Taylor (1993) is possible under a set of restrictions that generally do not hold within the theoretical model.

The representative agent economy is as in Benk et al (2008, 2010) but with a decentralized banking sector that produces credit as related to Gillman and Kejak (2011). By combining the business cycle with endogenous growth, stationary inflation lowers the output growth rate as supported empirically in Gillman et al. (2004) and Fountas et al. (2006), for example. Further, money supply shocks can cause inflation at low frequencies, as in Haug and Dewald (2012) and as supported by Sargent and Surico (2008, 2011), which can lead to output growth effects if the shocks are persistent and repeated. This allows shocks over the business cycle to cause changes in growth rates and in stationary ratios. The shocks to the goods sector productivity and the money supply growth rate are standard, while the third shock to credit sector productivity exists by virtue of the model's endogenous money velocity. Exchange credit is produced via a functional form used extensively in the financial intermediation microeconomics literature starting with Clark (1984) and promulgated by Berger and Humphrey (1997) and Inklaar and Wang (2013), for example.

[1] Since the Federal 2008 private banks have seen increasing regulation and central banks have seen pressure for greater oversight over their operations. The US Treasury bought private mortgage bonds in 2008 under President Bush's Emergency Economic Stabilization Act of 2008.such that these purchases so were subject to budget controls within the Congressional Budget Reconcilation Act. After Bush, the Federal Reserve System (Fed) instead bought mortgage loans directly and thereby skirted the Congressional budget process that Treasury purchases of the mortgage loans required. The fact the Fed need only remit profits to Treasury allows the Fed's private market purchases to have budget consequences delayed indefinitely.

[2] Taylor rule "reaction functions" have been motivated from various approaches including a quantity theoretic form of Fisher's (1922) equation of exchange. Taylor (1998, p.9) writes that "The policy rule is, of course, quite different from the quantity theory of money, but it is closely connected to the quantity equation. In fact, it can be easily derived from the quantity equation". Taylor notes that velocity depends on the interest rate, then writes down a version of the Taylor (1993) rule with the idea that it is derived directly from the quantity theory.

[3] Many different approaches to this broad topic, e.g. Davig and Leeper, (2007).

The shocks occur at the beginning of the period, are observed by the consumer before the decision making process commences, and follow a vector first-order autoregressive process. For goods sector productivity, z_t, the money supply growth rate, u_t, and bank sector productivity, v_t:

$$Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}, \tag{1}$$

where the shocks are $Z_t = [z_t \ u_t \ v_t]'$, the autocorrelation matrix is $\Phi_Z = diag\left\{\varphi_z, \varphi_u, \varphi_v\right\}$ and $\varphi_z, \varphi_u, \varphi_v \in (0,1)$ are autocorrelation parameters, and the shock innovations are $\varepsilon_{Zt} = [\varepsilon_{zt} \ \varepsilon_{ut} \ \varepsilon_{vt}]' \sim N(0, \Sigma)$. The general structure of the second-order moments is assumed to be given by the variance-covariance matrix Σ. These shocks affect the economy as described below, and as calibrated in Benk et al. (2010).

2.1 Consumer Problem

A representative consumer has expected lifetime utility from consumption of goods, c_t, and leisure, x_t; with $\beta \in (0,1)$, $\psi > 0$ and $\theta > 0$, this is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta \frac{(c_t x_t^{\psi})^{1-\theta}}{1-\theta}. \tag{2}$$

Output of goods, y_t, and increases in human capital, are produced with physical capital and effective labor each in Cobb-Douglas fashion; the bank sector produces exchange credit using labor and deposits as inputs. Let s_{Gt} and s_{Ht} denote the fractions of physical capital that the agent uses in goods production (G) and human capital investment (H), whereby:

$$s_{Gt} + s_{Ht} = 1. \tag{3}$$

The agent allocates a time endowment of one between leisure, x_t, labor in goods production, l_{Gt}, time spent investing in the stock of human capital, l_{Ht}, and time spent working in the bank sector (F subscripts for Finance), denoted by l_{Ft}:

$$l_{Gt} + l_{Ht} + l_{Ft} + x_t = 1. \tag{4}$$

Output of goods can be converted into physical capital, k_t, without cost and is thus divided between consumption goods and investment, denoted by i_t, net of capital depreciation. The capital stock used for production in the next period is therefore given by:

$$k_{t+1} = (1-\delta_k)k_t + i_t = (1-\delta_k)k_t + y_t - c_t. \tag{5}$$

The human capital investment is produced using capital $s_{Ht}k_t$ and effective labor $l_{Ht}h_t$, with $A_H > 0$ and $\eta \in [0,1]$, such that the human capital flow constraint is

$$h_{t+1} = (1-\delta_h)h_t + A_H (s_{Ht}k_t)^{1-\eta} (l_{Ht}h_t)^{\eta}. \tag{6}$$

With w_t and r_t denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents, $P_t w_t (l_{Gt} + l_{Ft}) h_t$ and $P_t r_t s_{Gt} k_t$, a nominal transfer from the government, T_t, and dividends from the bank. The consumer buys shares in the bank by making deposits of income at the bank. Each dollar deposited buys one share at a fixed price of one, and the consumer receives the residual profit of the bank as dividend income in proportion to the number of shares (deposits) owned. Denoting the real quantity of deposits by d_t, and the dividend per unit of deposits as R_{Ft}, the consumer receives a nominal dividend income of $P_t R_{Ft} d_t$. The consumer also pays to the bank a fee for credit services, whereby one unit of credit service is required for each unit of credit that the bank supplies the consumer for use in buying goods. With P_{Ft} denoting the nominal price of each unit of credit, and q_t the real quantity of credit that the consumer can use in exchange, the consumer pays $P_{Ft}q_t$ in credit fees.

With other expenditures on goods, of $P_t c_t$, and physical capital investment, $P_t k_{t+1} - P_t(1-\delta_k)k_t$, and on investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds $B_{t+1} - B_t(1+R_t)$, where R_t is the net nominal interest rate, the consumer's budget constraint is:

$$\begin{aligned} & P_t w_t \left(l_{Gt} + l_{Ft}\right) h_t + P_t r_t s_{Gt} k_t + P_t R_{Ft} d_t + T_t \\ \geq \quad & P_{Ft} q_t + P_t c_t + P_t k_{t+1} - P_t(1-\delta_k)k_t + M_{t+1} - M_t \\ & + B_{t+1} - B_t(1+R_t). \end{aligned} \tag{7}$$

The consumer can purchase goods by using either money M_t or credit services. With the lump sum transfer of cash T_t coming from the government at the beginning of the period, and with money and credit equally usable to buy goods, the consumer's exchange technology is:

$$M_t + T_t + P_t q_t \geq P_t c_t. \tag{8}$$

Since all cash comes out of deposits at the bank and credit purchases are paid off at the end of the period out of the same deposits, total deposits are equal to consumption. This gives the constraint that:

$$d_t = c_t. \tag{9}$$

Given k_0, h_0, and the evolution of M_t ($t \geq 0$) as given by the exogenous monetary policy in equation (17) below, the consumer maximizes utility subject to the budget, exchange and deposit constraints (8)-(9).

2.2 Banking Firm Problem

The bank produces credit that is available for exchange at the point of purchase. The bank determines the amount of such credit by maximizing its dividend profit subject to the labor and deposit costs of producing the credit. The production of credit uses a constant returns to scale technology with effective labor and deposited funds as inputs. In particular, with $A_F > 0$ and $\gamma \in (0,1)$:

$$q_t = A_F e^{v_t} \left(l_{Ft} h_t\right)^{\gamma} d_t^{1-\gamma}, \tag{10}$$

where $A_F e^{v_t}$ is the stochastic factor productivity.

Subject to the production function in equation (10), the bank maximizes profit Π_{Ft} with respect to the labor l_{Ft} and deposits d_t:

$$\Pi_{Ft} = P_{Ft} q_t - P_t w_t l_{Ft} h_t - P_t R_{Ft} d_t. \tag{11}$$

Equilibrium implies that:

$$\left(\frac{P_{Ft}}{P_t}\right) \gamma A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma - 1} = w_t; \tag{12}$$

$$\left(\frac{P_{Ft}}{P_t}\right) (1-\gamma) A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma} = R_{Ft}. \tag{13}$$

These indicate that the marginal cost of credit, $\left(\frac{P_{Ft}}{P_t}\right)$, is equal to the marginal factor price divided by the marginal factor product, or $\frac{w_t}{\gamma A_F e^{v_t} \left(\frac{l_{Ft} h_t}{d_t}\right)^{\gamma-1}}$, and that the zero profit dividend yield paid on deposits is equal to the fraction of the marginal cost given by $\left(\frac{P_{Ft}}{P_t}\right)(1-\gamma)\left(\frac{q_t}{d_t}\right)$.

2.3 Goods Producer Problem

The firm maximizes profit given by $y_t - w_t l_{Gt} h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in effective labor and capital:

$$y_t = A_G e^{z_t} (s_{Gt} k_t)^{1-\alpha} (l_{Gt} h_t)^{\alpha}. \tag{14}$$

The first order conditions for the firm's problem yield the standard expressions for the wage rate and the rental rate of capital:

$$w_t = \alpha A_G e^{z_t} \left(\frac{s_{Gt} k_t}{l_{Gt} h_t} \right)^{1-\alpha}, \tag{15}$$

$$r_t = (1-\alpha) A_G e^{z_t} \left(\frac{s_{Gt} k_t}{l_{Gt} h_t} \right)^{-\alpha}. \tag{16}$$

2.4 Government Money Supply

It is assumed that government policy includes sequences of nominal transfers as given by:

$$T_t = \Theta_t M_t = (\Theta^* + e^{u_t} - 1) M_t, \qquad \Theta_t = [M_t - M_{t-1}]/M_{t-1}, \tag{17}$$

where Θ_t is the growth rate of money and Θ^* is the stationary gross growth rate of money.

2.5 Definition of Competitive Equilibrium

The representative agent's optimization problem can be written recursively as:

$$V(s) = \max_{c,x,l_G,l_H,l_F,s_G,s_H,q,d,k',h',M'} \left\{ u(c,x) + \beta EV(s') \right\} \tag{18}$$

subject to the conditions (3) to (9), where the state of the economy is denoted by $s = (k,h,M,B;z,u,v)$ and a prime (') indicates next-period values. With the consumer's first-order conditions given in Appendix D, a competitive equilibrium consists of a set of policy functions $c(s)$, $x(s)$, $l_G(s)$, $l_H(s)$, $l_F(s)$, $s_G(s)$, $s_H(s)$, $q(s)$, $d(s)$, $k'(s)$, $h'(s)$, $M'(s)$, $B'(s)$ pricing functions $P(s)$, $w(s)$, $r(s)$, $R_F(s)$, $P_F(s)$ and a value function $V(s)$, such that:

i. the consumer maximizes utility, given the pricing functions and the policy functions, so that $V(s)$ solves the functional equation (18);

ii. the goods producer maximizes profit similarly, with the resulting functions for w and r being given by equations (15) and (16);

iii. the bank firm maximizes profit similarly in equation (11) subject to the technology of equation (10)

iv. the goods, money and credit markets clear, in equations (8) and (14), and in (8), (17), and (10).

3 General Equilibrium Taylor Condition

The 'Taylor condition' is now derived as an equilibrium condition of the Benk et al. (2010) model described in the previous section. Beginning from the first-order conditions of the model, we obtain:

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\theta} x_{t+1}^{\psi(1-\theta)}}{c_t^{-\theta} x_t^{\psi(1-\theta)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\}, \tag{19}$$

where R and π are gross rates of nominal interest and inflation, respectively. The term $\tilde{R}_t$ represents (one plus) a 'weighted average cost of exchange' as follows:

$$\tilde{R}_t = 1 + \frac{m_t}{c_t}(R_t - 1) + \gamma \left(1 - \frac{m_t}{c_t}\right)(R_t - 1),$$

where a weight of $\frac{m}{c}$ is attached to the opportunity cost of money $(R_t - 1)$ and a weight of $(1 - \frac{m}{c})$ is attached to the average cost of credit, $\gamma(R_t - 1)$, and $\frac{m_t}{c_t}$ is the real consumption normalised demand for money (i.e. the inverse of the consumption velocity of money). In effect, equation (19) augments a standard consumption Euler equation with the (growth rate of) the weighted average cost of exchange. If all goods purchases are conducted using money $(m_t/c_t = 1)$ then equation (19) reverts back to the familiar consumption Euler equation which would constitute an equilibrium condition of a standard, unit velocity cash-in-advance model without a money alternative.

For any variable z_t, define $\widehat{z}_t \equiv \ln z_t - \ln z$, where the absence of a time subscript denotes a *BGP* stationary value, and define $\widehat{g}_{z,t+1} \equiv \ln z_{t+1} - \ln z_t$, which approximates the growth rate at time $t+1$ for sufficiently small z_t. Consider a log-linear approximation of (19) evaluated around the *BGP*:

$$0 = E_t \left\{ \theta \widehat{g}_{c,t+1} - \psi(1-\theta)\widehat{g}_{x,t+1} + \widehat{g}_{\tilde{R},t+1} - \widehat{R}_{t+1} + \widehat{\pi}_{t+1} \right\}.$$

Rearranging this in terms of $\widehat{R}_t$ gives the Taylor condition expressed in log-deviations from the *BGP* equilibrium:

$$\widehat{R}_t = E_t \left\{ \Omega\widehat{\pi}_{t+1} + \Omega\theta\widehat{g}_{c,t+1} - \Omega\psi(1-\theta)\widehat{g}_{x,t+1} + (\Omega - 1)(R-1)\frac{\frac{m}{c}}{1-\frac{m}{c}}\widehat{g}_{\frac{m}{c},t+1} - (\Omega - 1)\widehat{R}_{t+1} \right\} \tag{20}$$

where

$$\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1. \tag{21}$$

The Taylor condition (20) can now be expressed in net rates (denoted by over-barred terms) and absolute deviations from the *BGP* equilibrium, as demonstrated by the following proposition.

Proposition 1 *An equilibrium condition of the economy takes the form of a Taylor Rule which sets deviations of the short-term nominal interest rate from some baseline path in proportion to deviations of variables from their targets:*

$$\begin{aligned} \overline{R}_t - \overline{R} \quad = \quad & \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega\psi(1-\theta)E_t\overline{g}_{x,t+1} \\ & + \frac{(\Omega-1)\overline{R}\frac{m}{c}}{1-\frac{m}{c}}E_t\overline{g}_{\frac{m}{c},t+1} - (\Omega-1)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{22}$$

where $\Omega \geq 1$, and for a given w, then $\frac{\partial\Omega}{\partial R} > 0$ and $\frac{\partial\Omega}{\partial A_F} > 0$, and the target values are equal to the balanced growth path equilibrium values.[4]

Proof. Since the *BGP* solution for normalized money demand is:

$$0 \leq \frac{m}{c} = 1 - A_F\left(\frac{\overline{R}\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}} \leq 1, \tag{23}$$

then $\Omega \equiv 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left(1+\overline{R}\right)\left[\gamma+\frac{m}{c}(1-\gamma)\right]} \geq 1$ and, given w, $\frac{\partial\Omega}{\partial R} \geq 0$ and $\frac{\partial\Omega}{\partial A_F} \geq 0$. ■

Corollary 2 *An increase in the BGP money supply growth rate Θ^*, for the log-utility case of $\theta = 1$, cause the BGP nominal interest rate $\overline{R}$ to increase, and so causes Ω to increase.*

Proof. From the cash-in-advance constraint (8), and equations (17), (19) and (23) for $\theta = 1$, the money supply growth rate after the lump sum transfer equals the growth rate of the nominal price level plus the rate of time preference: $\overline{R} = \Theta^* + \rho$. If Θ^* increases, then $\overline{R}$ increases, and so by the preceding proposition Ω increases. ■

For a linear production function of goods w is the constant marginal product of labor but more generally w is endogenous and will change; however this change in w is quantitatively small compared to changes in R and A_F, so that the derivatives above almost always hold true. Note that for a unitary consumption velocity of money, the velocity growth and forward interest terms drop out of equation (22)

The term $\overline{\pi}$ in equation (22) can be compared to the inflation target that features in many interest rate rules (e.g. Taylor, 1993; Clarida et al., 2000). This is usually set as an exogenous constant in a conventional rule but represents the *BGP* rate of inflation in the Taylor condition.[5] The term in consumption growth is similar, but not identical to, the first difference of the output gap that features in the so-called ‘speed limit’ rule (Walsh, 2003). Alternatively, the term in the growth rate of leisure time can be compared to the unemployment rate which sometimes features in conventional interest rate rules in place of the output gap.[6]

Equation (22) also contains two terms which are not usually found in standard monetary policy reaction functions. Firstly, there is a term in the growth rate of the real (consumption normalized) demand for money. Conventional interest rate rules are usually considered in the context of models which omit monetary relationships and thus money demand does not feature directly in the model.[7] Secondly, the Taylor condition contains a term in the expected future nominal interest rate. This contrasts with the lagged nominal interest term which is often used to capture ‘interest rate

[4]This is the the Brookings project form of the Taylor rule as described in Orphanides (2008).

[5]Although see Ireland (2007) for an example of a conventional interest rate rule with a time-varying inflation target.

[6]For example, Mankiw (2001) includes the unemployment rate in an interest rate rule and Rudebusch (2009) includes the ‘unemployment gap’.

[7]Specifically, shifts in the demand for money are perfectly accommodated by adjustments to the money supply in order to maintain the rule-implied nominal interest rate. This, it is claimed, renders the evolution of the money supply an operational detail which need not be modelled directly (e.g. Woodford, 2008).

smoothing' in a conventional rule (e.g. Clarida et al., 2000). Other interpretations in terms of a backwards looking rule and a credit interpretation are given in Appendix A.

In general, the coefficient on inflation in equation (22) exceeds unity $(\Omega > 1)$. This replicates the 'Taylor principle' whereby the nominal interest rate responds more than one-for-one to (expected future) inflation deviations from 'target'. However, the inflation coefficient in the Taylor condition does not reflect policy-makers' preferences. Rather, it is a function of the *BGP* nominal interest rate (R), the consumption normalized demand for real money balances (m/c) and the efficiency with which the banking sector transforms units of deposits into units of the credit service, as reflected by the magnitude of $(1-\gamma)$. Furthermore, higher productivity in the banking sector (A_F) causes a higher velocity and implies a larger inflation coefficient in the Taylor condition. The magnitude of Ω clearly does not reflect a response to inflation in the conventional 'reaction function' sense.[8]

Equation (22) can alternatively be rewritten in terms of the consumption velocity of money, $V_t \equiv \frac{c_t}{m_t}$, and the productive time, or 'employment', growth rate ($l \equiv l_G + l_H + l_F = 1 - x$). Using the fact that $\widehat{x}_t = -\frac{1-x}{x}\widehat{l}_t$:

$$\begin{aligned}\overline{R}_t - \overline{R} \quad = \quad & \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t \left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ & -\Omega_V E_t \overline{g}_{V,t+1} - \left(\Omega - 1\right) E_t \left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{24}$$

Where over-barred terms again denote net rates and:

$$\Omega_V \equiv \frac{\left(\overline{R}\right)}{1+\overline{R}}\left(\frac{(1-\gamma)\frac{m}{c}}{\gamma + (1-\gamma)\frac{m}{c}}\right).$$

Proposition 3 *For the Taylor condition of equation (24), it is always true that* $0 \leq \Omega_V \leq 1 \leq \Omega$.

Proof.

$$\begin{aligned}\Omega \quad & \equiv \quad 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left(1+\overline{R}\right)\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1;\ \frac{m}{c} = 1 - A_F^{\frac{1}{1-\gamma}}\left[\frac{\overline{R}\gamma}{w}\right]^{\frac{\gamma}{1-\gamma}} \leq 1; \\ 1 \quad & \geq \quad (1-\gamma)\left(1-\frac{m}{c}\right) \geq 0;\ \Rightarrow 0 \leq \Omega_V \equiv \frac{\overline{R}(1-\gamma)}{1+\overline{R}}\left(\frac{\frac{m}{c}}{1-(1-\gamma)\left(1-\frac{m}{c}\right)}\right) \leq 1; \\ & \Rightarrow \quad 0 \leq \Omega_V \leq 1 \leq \Omega. \end{aligned}$$

■

At the Friedman (1969) optimum $(\overline{R} = 0)$, $\frac{m}{c} = 1$, $\omega = 0$, and the velocity coefficient (Ω_V) takes a value of zero. The velocity growth term only enters the Taylor condition when the nominal interest rate differs from the Friedman (1969) optimum and fluctuates. In turn, this has implications for $\Omega = 1 + \left(\frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left(1+\overline{R}\right)\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right)$,since when $\overline{R} = 0$, $(1-\gamma)\left(1-\frac{m}{c}\right) = 0$, and $\Omega = 1$. For $\frac{m}{c}$ below one (velocity above one), which is true for most practical experience, the model's equivalent of the 'Taylor principle' $(\Omega > 1)$ holds.

Corollary 4 *Given* w, *then* $\frac{\partial\Omega}{\partial\overline{R}} \geq 0$, $\frac{\partial\Omega_V}{\partial\overline{R}} \geq 0$, $\frac{\partial\Omega}{\partial A_F} \geq 0$, $\frac{\partial\Omega_V}{\partial A_F} \leq 0$.

Proof. This comes directly from the definitions of parameters above. ■

A higher target nominal interest rate can be accomplished only by a higher *BGP* money supply growth rate. This would in turn make the inflation and consumption growth coefficients larger and the forward interest rate and velocity coefficients would become more negative. A higher credit productivity factor A_F, and so a higher velocity, leads to a higher inflation coefficient and a more negative response to the forward-looking interest term but a less negative coefficient on the velocity growth term.

[8] Unlike Sørensen and Whitta-Jacobsen's (2005, pp.502-505) quantity theory based equilibrium condition, the inflation coefficient in (22) exceeds unity for any (admissible) interest elasticity of money demand. In their expression, the inflation coefficient falls below unity if the interest (semi) elasticity of money demand exceeds one in absolute value. In the Benk et al. (2010) model, the coefficient on inflation would exceed unity even in this case but the central bank would not wish to increase the money supply growth rate to this extent because seigniorage revenues would begin to recede as the elasticity increases beyond this point.

The Taylor condition above would look identical with exogenous growth. However, under exogenous growth the targeted inflation rate and growth rate of the economy are unrelated and exogenously specified. Under endogenous growth the targets are instead the endogenously determined *BGP* values for inflation, the growth rate, and the nominal interest rate and each of these are determined, in part, by the long run stationary money supply growth rate Θ^*, which is exogenously given. In turn, Θ^* translates directly into a long run inflation target accepted by the central bank, such as the two percent target often incorporated into conventional interest rate rules (for example, Taylor, 1993). So the model assumes only a long run money supply growth target, or alternatively, a long run inflation rate target.

3.1 Misspecified Taylor Condition with Output Growth

It is not surprising to find that the growth rate of consumption appears in equation (24) rather than the output growth rate given that the derivation of the Taylor condition begins from the consumption Euler equation (19). However, the Taylor condition can be rewritten to include an output growth term and hence correspond more closely to standard Taylor rule specifications, in particular the 'speed limit' rule considered by Walsh (2003). To derive this alternative rule, consider that the identity $y_t = c_t + i_t$ implies that $\widehat{y_t} = \frac{c}{y}\widehat{c_t} + \frac{i}{y}\widehat{i_t}$, where $\widehat{i_t} = \frac{k}{i}\left[\widehat{k_t} - (1-\delta)\widehat{k}_{t-1}\right]$. The growth rate of investment can be understood as the acceleration of the growth of capital gross of depreciation. The Taylor condition can be rewritten as:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[\frac{y}{c}E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) - \frac{i}{c}E_t\left(\overline{g}_{i,t+1} - \overline{g}\right)\right] \\ &\quad + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{25}$$

A term in investment growth does not appear in standard Taylor rules but plays a role as part of what is interpreted as the output gap growth rate in this modified Taylor condition. Equation (25) forms the basis for the two misspecified estimating equations considered in Section 5. The first misspecified estimating equation simply replaces the consumption growth term in equation (24) with an output growth term as follows:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \\ &\quad + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \Omega_V E_t\overline{g}_{V,t+1} - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{26}$$

Comparing equation (25) and equation (26) shows that the latter erroneously overlooks the weighting on the output growth rate ($\frac{y}{c}$) and omits the term in the investment growth rate. Replacing consumption growth with output growth without the additional term in investment therefore misrepresents the structure of the underlying Benk et al. (2010) model and as such equation (26) is misspecified. Note that with no physical capital in the economy, equation (26) would be a valid equilibrium condition of the economy.

3.2 Misspecified Standard Taylor Rule

The second misspecified model erroneously imposes the same restrictions used to arrive at equation (26) but also drops the terms in productive time and velocity, giving:

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta\left[E_t\left(\overline{g}_{y,t+1} - \overline{g}\right)\right] \\ &\quad - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{27}$$

This can be interpreted as a conventional interest rate rule with a forward-looking 'interest rate smoothing' term; the additional restriction that $\Omega = 1$ would replicate a standard interest rate rule without interest rate smoothing. Once again, equation (27) does not accurately represent an equilibrium condition of the Benk et al. (2010) economy and is therefore misspecified. Equation (27) with $\Omega = 1$ would be the correct equilibrium condition if the economy featured neither physical capital nor exchange credit.

Preferences		
θ	1	Relative risk aversion parameter
ψ	1.84	Leisure weight
β	0.96	Discount factor
Goods Production		
α	0.64	Labor share in goods production
δ_k	0.031	Depreciation rate of goods sector
A_G	1	Goods productivity parameter
Human Capital Production		
ε	0.83	Labor share in human capital production
δ_h	0.025	Depreciation rate of human capital sector
A_H	0.21	Human capital productivity parameter
Banking Sector		
γ	0.11	Labor share in credit production
A_F	1.1	Banking productivity parameter
Government		
σ	0.05	Money growth rate

Table 1: Parameters

g	0.024	Avg. annual output growth rate
π	0.026	Avg. annual inflation rate
R	0.0944	Nominal interest rate
l_G	0.248	Labor used in goods sector
l_H	0.20	Labor used in human capital sector
l_F	0.0018	Labor used in banking sector
i/y	0.238	Investment-output ratio in goods sector
m/c	0.38	Share of money transactions
x	0.55	Leisure time
$l \equiv 1-x$	0.45	Productive time

Table 2: Target Values

4 Calibration

We follow Benk et al. (2010) in using postwar U.S. data to calibrate the model (Table 1) and calculate a series of 'target values' consistent with this calibration (Table 2); see Benk et al. for the shock process calibration.

Subject to this calibration, we derive a set of theoretical 'predictions' for the coefficients of the Taylor condition (24). These values will subsequently be compared to the coefficients estimated from artificial data simulated from the model. Consider first the inflation coefficient (Ω). According to the calibration and target values presented in tables 1 and 2, its theoretical value is

$$\Omega = 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} = 1 + \frac{(1-0.11)\,(1-0.38)}{1.0944\,(1-(1-0.11)\,(1-0.38))} = 2.125$$

For $R=1$, only cash is used so that $\frac{m}{c}=1$ and Ω reverts to its lower bound of 1. This also happens with zero credit productivity $(A_F=0)$, in which case only cash is used in exchange.

The remaining coefficients, except for velocity, are simple functions of the inflation coefficient. The consumption growth coefficient is $\Omega\theta$, which with $\theta=1$ for log-utility should simply take the same magnitude as the coefficient on inflation $(\theta\Omega = 2.125)$. The coefficient on the productive time growth rate should take a value of zero with log utility. However with leisure preference calibrated at 1.84, and productive time $(1-x \equiv l)$ equal to equal to 0.45 along the *BGP*, the estimated value of the productive time coefficient can be interpreted as implying a certain θ factored by $\Omega\psi\frac{l}{1-l} = (2.125)(1.84)\frac{0.45}{0.55} = 3.199$. Given the magnitude of the inflation coefficient, the coefficient on the forward interest term is simply $-(\Omega-1) = -1.125$. The velocity coefficient $(-\Omega_V)$ is -0.065 using:

$$-\frac{(R-1)}{R}\left(\frac{(1-\gamma)\frac{m}{c}}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right) = -\frac{(1.0944-1)}{1.0944}\left(\frac{(1-0.11)\,0.38}{(1-(1-0.11)(1-0.38))}\right).$$

At the Friedman (1969) optimum $(R=1)$, $\Omega_V = 0$. In this case the omission of the term in velocity growth in the estimation exercises that follow would be innocuous but this is not true in general.

5 Artificial Data Generation and Estimation

The Benk et al. (2010) model presented in Section 2 is simulated using the calibration provided in Table 1 in order to generate 1000 alternative 'joint histories' for each of the variables in equation (24), where each history is 100 periods in length. To do so, 100 random sequences for the shock vector innovations are generated and control functions of the log-linearized model are used to compute sequences for each variable. Each observation within a given history may be thought of as an annual period given the frequency considered by the Benk et al. (2010) model. The data set used to estimate the coefficients of the Taylor condition can therefore be viewed as comprising of 1000, '100-year', samples of artificial data.

5.1 Estimation Methodology

This section presents the results of estimating a 'correctly specified' estimating equation based upon the true theoretical relationship (24) against artificial data generated from the Benk et al. (2010) model.[9] In a similar manner, two alternative estimating equations are evaluated using the same data set. Since these alternative estimating equations differ from the expression based upon the true theoretical relationship, they necessarily constitute misspecified empirical models.[10]

Prior to estimation, the simulated data is filtered by either 1) a Hodrick-Prescott (HP) filter with a smoothing parameter selected in accordance with Ravn and Uhlig (2002); 2) a $3-8$ period ("year") Christiano and Fitzgerald (2003) band pass filter for 'business cycle frequencies'; or 3) a $2-15$ year Christiano and Fitzgerald (2003) band pass filter which retains more of the lower frequency trends in the data than the $3-8$ year filter, in the spirit of Comin and Gertler's (2006) 'medium-term cycle'.[11] *A priori*, the $2-15$ band pass filter might be regarded as the 'most relevant' to the underlying theoretical model because shocks in the model can cause low frequency events during the business cycle, such as a change in the permanent income level without a reversion to its previous level.[12]

The first estimation technique considered is OLS, as used by Taylor (1999) in the context of a contemporaneous interest rate rule. However, if expected future variables are correlated with the error term then a suitable set of instruments are required to proxy for these forward-looking terms.[13] Two instrumental variables (IV) techniques are considered and each differs by the instrument set employed. The first is a two stage least squares (2SLS) estimator under which the first lags of inflation, consumption growth, productive time growth and velocity growth and the second lag of the nominal interest rate are used as instruments. Adding a constant term to the instrument set provides a 'just identified' 2SLS estimator. In using lagged variables as instruments we exploit the fact that such terms are

[9] The exercise conducted here is similar to those conducted by Fève and Auray (2002), for a standard CIA model, and Salyer and Van Gaasbeck (2007), for a 'limited participation' model.

[10] We acknowledge that in a full information maximum likelihood estimation that uses all of the equilibrium conditions of the economy we may be able to recover the theoretical coefficients of the Taylor condition almost exactly; we leave that exercise as an important part of future research that encompasses the entire alternative model; and then we could also compare it to the standard three equation central bank policy model.

[11] However, Comin and Gertler's 'medium-term cycle' is defined using a wider 2-200 quarter filter. However, the 2-15 filter will still retain periodicities that the HP and 3-8 filters consign to the 'trend'.

[12] In principle, the filtering procedure takes account of the Siklos and Wohar (2005) critique of empirical Taylor rule studies which do not address the non-stationarity of the data. However, standard ADF and KPSS tests suggest that the simulated data is stationary prior to filtering (results not reported). Accordingly, the filters do not implement a de-trending procedure.

[13] Empirical studies usually deal with expected future terms either byreplacing them with realised future values and appealing to rationalexpectations for the resulting conditional forecast errors (e.g. Clarida et al., 1998, 2000) or by using private sector or central bank forecasts as empirical proxies (e.g. Orphanides, 2001; Siklos and Wohar, 2005).

pre-determined and thus not susceptible to the simultaneity problem which motivates the use of IV techniques. The 2SLS procedure applies a Newey-West adjustment for heteroskedasticity and autocorrelation (HAC) to the coefficient covariance matrix.

The second IV procedure is a generalized method of moments (GMM) estimator under which three additional lags of inflation, consumption growth, productive time growth and velocity growth and two further lags of the nominal interest rate are added to the instrument set.[14] Expanding the instrument set in this manner reduces the sample size available for each of the 1000 simulated sample periods but the over-identifying restrictions can now be used to test the validity of the instrument set using the Hansen J-test. The GMM estimator used iterates on the weighting matrix in two steps and applies a HAC adjustment to the weighting matrix using a Bartlett kernel with a Newey-West fixed bandwidth.[15] A similar HAC adjustment is also applied to the covariance weighting matrix.

The results are presented in three sets of tables, one set for each estimating equation, and are further subdivided according to the statistical filter applied to the simulated data. Alongside the estimates obtained from an 'unrestricted' estimating equation, each table also reports estimates derived from a 'restricted' estimating equation which arbitrarily omits the forward interest rate term $(\beta_5 = 0)$. This arbitrary restriction demonstrates the importance of the dynamic term in equation (24). Each table of results reports mean coefficient estimates along with the standard error of these estimates (as opposed to the mean standard error). The figures in square brackets report the number of coefficients estimated to be statistically different from zero at the 5% level of significance and this count is used as an indication of the 'precision' of the estimates. An 'adjusted mean' figure is also reported for each coefficient; this is obtained by setting non statistically significant coefficient estimates to zero when calculating the averages. The tables also report mean R-square and mean adjusted R-square statistics along with the mean P-value for the F-statistic for overall significance (these cannot be computed for the GMM estimator), the mean P-value for the Hansen J-statistic which tests the validity of the instrument set (these can only be calculated in the presence of over-identifying restrictions), and the mean Durbin-Watson (D-W) statistic which tests for autocorrelation. The number of estimations for which the null hypothesis of the J-statistic is *not* rejected - i.e. the instrument set is not found to be invalid - is reported alongside its mean P-value and the number of simulated series for which the D-W statistic exceeds its upper critical value - i.e. the null hypothesis that the residuals are serially uncorrelated cannot be rejected - is reported alongside the mean D-W statistic.[16]

5.2 General Taylor Condition

Tables 3-6 present estimates obtained from the following 'correctly specified' estimating equation:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{c,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \tag{28}$$

Expected future variables on the right hand side are obtained directly from the model simulation procedure and are instrumented for as described above.

The key result is that using a $2-15$ year window Table 3 consistently reports an inflation coefficient which exceeds unity for the estimating equation which accurately reflects the underlying theoretical model. This result is found to be robust to the statistical filter applied to the data and to the estimator employed, subject to the estimator providing a 'precise' set of estimates. See Appendix B for Tables B-1 and B-2 which report similar results using instead HP filtered and a $3-8$ year business cycle window. The forward interest rate term is also found to be important in terms of generating a coefficient on inflation consistent with the underlying Benk et al. (2010) model. Arbitrarily omitting this dynamic term yields much smaller estimates for the inflation coefficient to the extent that the mean estimate often falls below unity.

[14] Carare and Tchaidze (2005, p.15) note that the four-lags-as-instruments specification is the standard approach in the interest rate rule literature (e.g. Orphanides, 2001). Although the GMM procedure in general corrects for autocorrelation and heteroskedasticity, in estimating with simulated data we use lags as 'valid' instruments for pre-determined variables. These instruments might prove to be 'relevant' because the data is serially correlated but no further lags are needed for the estimating equation itself. For actual data, Clarida et al. (QJE, 2000, p.153) use a GMM estimator "with an optimal weighting matrix that accounts for possible serial correlation in [the error term]" but they also add two lags of the dependent variable to their estimating equation on the basis that this "seemed to be sufficient to eliminate any serial correlation in the error term." (p.157), implying that the GMM correction was insufficient for this purpose.

[15] Jondeau et al. (2004, p.227) state that: "To our knowledge, all estimations of the forward-looking reaction function based on GMM have so far relied on the two-step estimator." They proceed to consider more sophisticated GMM estimators but nevertheless identify advantages to the "simple approach" (p.238) adopted in the literature.

[16] The D-W count excludes cases for which the test statistic falls in the inconclusive region of the test's critical values.

In terms of the general features of the results obtained from the unrestricted specification, the OLS and GMM procedures tend to generate a greater number of statistically significant estimates than the 2SLS estimator. Table 3 shows that the 2SLS estimator provides a statistically significant estimate for the inflation coefficient for only 580 of the 1000 simulated histories while the OLS and GMM estimators both return 1000 statistically significant estimates. The OLS and GMM procedures generate reasonably large R-square and adjusted R-square statistics, whereas negative R-square statistics are obtained from the simple 2SLS estimator. Expanding the instrument set in order to implement the GMM procedure leads to 1000 rejections of the J-test for instrument validity across all three filters. One might also be wary of the high number of D-W null hypothesis rejections produced by the OLS estimator, although the mean D-W statistic remains 'reasonably large' in each case; 1.56 for the $2-15$ filter, for example.[17]

Table 3 reports that the mean estimate for the inflation coefficient is 2.179 using the OLS estimator and 2.306 using the GMM estimator.[18] These estimates compare favorably to the theoretical prediction of $\Omega = 2.125$. The right hand side of Table 3 shows that the mean estimate of the inflation coefficient falls below unity for the OLS and GMM estimators when the forward interest rate term is arbitrarily omitted from the estimating equation. A precise mean estimate of 0.614 is obtained from the OLS estimator and a similarly precise mean estimate of 0.964 is obtained from the GMM procedure. Similar OLS and GMM estimates are obtained for the inflation coefficient under the two alternative filters in Appendix B, Tables B-1 and B-2, both in terms of the mean coefficient estimates for the unrestricted specification and in terms of the decline in magnitude induced by the arbitrary restriction.

In contrast to the estimated inflation coefficients, the estimated coefficients for consumption growth and productive time growth diverge from their theoretical predictions for the 'unrestricted' estimating equation. Under log utility ($\theta = 1$), the former should take the same magnitude as the coefficient on inflation and the latter should take a value of zero. The coefficient estimates can be used to 'back-out' an estimate of the coefficient of relative risk aversion (θ). Firstly, using the mean GMM estimate for the coefficient on consumption growth of 0.302 (Table 3) and the corresponding estimate of Ω, an implied estimate of θ can be calculated as $\frac{\beta_2}{\beta_1} = \frac{0.302}{2.306} = 0.131$, which is substantially smaller than the baseline calibration of $\theta = 1$. Alternatively, the relationship $\beta_3 = \beta_1 \psi (1-\theta) l/(1-l)$, which is obtained from equation (24) with Ω replaced by its estimate β_1, can also be used to obtain an implied estimate of θ. Using the estimates presented in Table 3, the implied estimate would be $\theta = 1.103$, which is much closer to the calibrated value.

Table 3 also reports that both the OLS and GMM procedures generate 1000 statistically significant estimates for the coefficient on velocity growth under the unrestricted estimating equation and that the mean estimate is correctly signed for both estimators. The mean coefficient estimates are reported as -0.196 and -0.269 for OLS and GMM estimators respectively; these estimates are somewhat smaller than the theoretical prediction of -0.065. Similar estimates are obtained under the HP and $3-8$ filters. Finally, Table 3 reports mean estimates of -1.761 (OLS) and -1.729 (GMM) for the forward interest rate coefficient compared to a theoretical prediction of -1.125. The mean estimates are therefore correctly signed but, again, smaller than the theoretical prediction.

In a standard interest rate rule an inflation coefficient in excess of unity is interpreted to reflect policy-maker's dislike of inflation deviations from target. However, this interpretation is not applicable to the Taylor condition. The result that the coefficient on inflation exceeds unity is a consequence of a money growth rule not an interest rate rule. Similarly, the break-down of the Taylor principle under the 'restricted' estimating equation ($\beta_5 = 0$) cannot be interpreted as a softening of policy-makers' attitude towards inflation; this result simply emanates from model misspecification.

5.3 Taylor Condition with Output Growth

The same estimation procedure is now applied to an estimating equation which replaces consumption growth in equation (28) with output growth as follows:

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \tag{29}$$

[17] The results for the $3-8$ band pass filter in Table B-2 are unusual in this sense in that all three estimation procedures produce a high number of D-W test rejections. For the other two filters, this undesirable result is confined to the OLS estimator.

[18] The discussion focuses on the OLS and GMM estimators because they produce more 'precise' estimates and also because the OLS estimator tends to reject the null hypothesis of the F-statistic more frequently than the 2SLS estimator (1000 vs. 907 rejections in Table 3, for example). The OLS regressions are possibly afflicted by autocorrelation however, as discussed above, thus one might favor the GMM estimates.

BP Filter, 2-15 Window	Unrestricted			Assumed $\beta_5 = 0$		
	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-2.26E-06 [0]	4.22E-05 [0]	2.54E-07 [22]	-1.82E-06 [0]	-2.46E-06 [0]	3.44E-06 [21]
Standard error	4.01E-05	0.001	4.82E-05	3.27E-05	4.83E-05	6.26E-05
Adjusted mean	-	-	4.25E-07	-	-	9.03E-07
$E_t\pi_{t+1}$	2.179 [1000]	3.816 [580]	2.306 [1000]	0.614 [999]	1.127 [763]	0.964 [999]
Standard error	0.195	51.040	0.272	0.108	0.640	0.169
Adjusted mean	2.179	1.402	2.306	0.614	0.936	0.963
$E_t g_{c,t+1}$	0.277 [1000]	0.570 [730]	0.302 [1000]	0.170 [1000]	0.262 [851]	0.207 [1000]
Standard error	0.016	5.546	0.025	0.017	0.103	0.026
Adjusted mean	0.277	0.265	0.302	0.170	0.230	0.207
$E_t g_{l,t+1}$	-0.295 [997]	-0.737 [526]	-0.359 [999]	-0.210 [732]	-0.263 [405]	-0.277 [935]
Standard error	0.067	8.208	0.085	0.088	0.203	0.111
Adjusted mean	-0.294	-0.242	-0.359	-0.182	-0.152	-0.272
$E_t g_{V,t+1}$	-0.196 [1000]	-0.347 [807]	-0.269 [1000]	-0.158 [998]	-0.307 [944]	-0.236 [1000]
Standard error	0.024	0.271	0.031	0.032	0.077	0.042
Adjusted mean	-0.196	-0.273	-0.269	-0.158	-0.292	-0.236
$E_t R_{t+1}$	-1.761 [1000]	-5.586 [335]	-1.729 [1000]			
Standard error	0.201	114.905	0.322			
Adjusted mean	-1.761	-0.712	-1.729			
$Mean$;						
R-square	0.830	<0	0.782	0.625	<0	0.522
Adjusted R-square	0.821	<0	0.770	0.609	<0	0.501
Pr(F-statistic)	2.50E-24 (1000)	0.051 (907)	N/A	5.04E-10 (1000)	0.003 (985)	N/A
Pr(J-statistic)	N/A	N/A	0.315 {1000}	N/A	0.298 {757}	0.298 {1000}
Durbin-Watson	1.558 $<141>$	2.059 $<972>$	2.040 $<881>$	1.954 $<864>$	2.052 $<998>$	2.205 $<977>$
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and $<>$ the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 3: Taylor Condition Estimation, Band Pass Filtered Data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

The simulated data remains unchanged, therefore equation (29) represents a misspecified version of the 'correct' estimating equation, which continues to be equation (28).[19] In particular, equation (29) can be seen to correspond to the misspecified Taylor condition, equation (26).

The results are similar across the HP, the $3-8$ band pass and the $2-15$ band pass filters but as the latter filter gives the most statistically significant coefficient estimates only estimates from the $2-15$ filter are presented in Table 4. Comparing the general features of the results to those presented in Table 3, there has been a decline in the precision with which the coefficients are estimated, a decline in the magnitude of the R-square and adjusted R-square statistics and a decline in the number of rejections of the null hypothesis of the F-statistic for joint significance. This is not surprising given that an element of misspecification has been introduced into the estimating equation. The number of rejections of the null hypothesis of the D-W test statistic also tends to increase although the GMM procedure applied to $2-15$ filtered data still fails to reject the null for 94.5% of the simulated samples.

The estimated inflation coefficients are now found to be substantially greater than the coefficients obtained from the 'correctly specified' estimating equation (28) and hence substantially greater than the predicted value. For instance, the GMM estimate for the unrestricted estimating equation rises from 2.306 in Table 3 to 5.274 in Table 4 (or 5.235 according to the adjusted mean). Similarly, the OLS estimate increases from 2.179 to 4.219 (or 4.185 adjusted).[20]. The estimates clearly diverge further from the theoretical prediction of $\Omega = 2.125$ under this particular form of misspecification.

The incorrectly specified estimating equation also induces a substantial decrease in the estimated coefficients for the productive time growth rate and the forward nominal interest rate. The estimated coefficient on the productive time growth rate decreases from -0.294 to -2.073 (both adjusted means) between Table 3 and Table 4 according to the OLS estimator and from -0.359 to -2.790 for the GMM estimator, hence the estimates diverge further from their predicted value of $\beta_3 = 0$. The GMM estimates of the forward interest rate term also decrease from -1.729 in Table 3 to -4.372 in Table 4 (adjusted means where appropriate). Again, the estimates diverge further from theoretical prediction of -1.125.

The estimated coefficients for output growth in Table 4 are comparable to those for consumption growth presented in Table 3, despite the effect that the misspecification has on the other estimates. For example, the OLS estimate for β_3 is 0.300 (adjusted mean) in Table 4 compared to the corresponding estimate of 0.277 in Table 3. For the GMM estimator the coefficient on output growth is 0.402 (adjusted mean) in Table 4 compared to the corresponding estimate of 0.302 reported in Table 3.

The velocity growth term is estimated precisely by the GMM estimator even after the modification to the estimating equation. Estimates of β_4 retain the correct sign and are of a similar magnitude as under the correctly specified estimating equation; for example, a GMM estimate of -0.190 in Table 4 compared to a corresponding estimate of -0.269 in Table 3.

For the restricted specification ($\beta_5 = 0$), the estimates undergo similar changes as those obtained from the restricted version of the 'correct' estimating equation (28). The OLS and GMM estimators generate inflation coefficients which often fall below unity in a manner incompatible with the theoretical model from which the Taylor condition is derived, although the GMM estimator provides a notable exception (Table 4).

In short, the results obtained from applying equation (29) to the simulated data show that adapting the estimating equation in a seemingly minor way can have a substantial impact upon the coefficient estimates obtained. The erratic results produced by this misspecified estimating equation provide an illustration of the fundamental difference between the Taylor condition and a conventional interest rate rule. Unlike a Taylor rule, the Taylor condition cannot be modified in an *ad hoc* manner.[21] In order to make the progression from (28) to (29) in a legitimate manner, one would need to alter the underlying model by excluding physical capital, for example. A new set of artificial data would then need to be simulated from this alternative model prior to re-estimation.

5.4 A Conventional Interest Rate Rule

The estimation procedure is now re-applied to the following estimating equation:

[19] The instrument sets used for the 2SLS and GMM estimators are modified by replacing consumption growth with output growth but remains unchanged in terms of the number of lags included.

[20] Corresponding upward shifts in the estimated inflation coefficient are found for the $3-8$ band pass filter results (results not reported) and even larger increases are found for the HP filtered data (results not reported).

[21] In contrast, conventional interest rate rules are exogenously specified and thus amenable to arbitrary modifications. Clarida et al. (1998), for example, add the exchange rate to the standard Taylor rule and Cecchetti et al. (2000) and Bernanke and Gertler (2001) consider whether policymakers should react to asset prices.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-3.66E-06 [0]	0.001 [0]	3.07E-06 [16]	-9.34E-07 [0]	-4.46E-06 [0]	2.80E-06 [5]
Standard error	6.19E-05	0.031	8.58E-05	2.43E-05	9.65E-05	6.42E-05
Adjusted mean	-	-	-9.38E-07	-	-	4.23E-07
$E_t \pi_{t+1}$	4.219 [971]	-25.003 [189]	5.274 [961]	0.541 [941]	2.338 [882]	1.101 [990]
Standard error	1.715	1290.303	2.582	0.170	1.151	0.264
Adjusted mean	4.185	2.522	5.235	0.532	2.010	1.100
$E_t g_{y,t+1}$	0.303 [967]	-2.019 [206]	0.406 [940]	0.038 [563]	0.211 [262]	0.082 [956]
Standard error	0.125	122.643	0.204	0.020	0.304	0.029
Adjusted mean	0.300	0.244	0.402	0.029	0.077	0.081
$E_t g_{l,t+1}$	-2.098 [959]	14.698 [211]	-2.815 [954]	-0.284 [424]	-1.462 [353]	-0.610 [941]
Standard error	0.892	825.197	1.417	0.189	1.740	0.232
Adjusted mean	-2.073	-1.672	-2.790	-0.189	-0.655	-0.598
$E_t g_{V,t+1}$	-0.118 [884]	0.069 [310]	-0.191 [970]	-0.095 [733]	-0.246 [587]	-0.158 [923]
Standard error	0.042	16.979	0.064	0.043	0.164	0.061
Adjusted mean	-0.113	-0.117	-0.190	-0.084	-0.161	-0.156
$E_t R_{t+1}$	-3.878 [907]	29.503 [128]	-4.498 [849]			
Standard error	1.812	1437.910	2.802			
Adjusted mean	-3.767	-1.757	-4.372			
$Mean;$						
R-square	0.361	<0	0.127	0.246	<0	<0
Adjusted R-square	0.327	<0	0.079	0.214	<0	<0
Pr(F-statistic)	0.001 (995)	0.379 (411)	N/A	0.020 (924)	0.055 (829)	N/A
Pr(J-statistic)	N/A	N/A	0.226 {1000}	N/A	0.260 {682}	0.264 {1000}
Durbin-Watson	1.882 <699>	1.982 <868>	2.342 <945>	2.295 <1000>	2.145 <997>	2.728 <999>
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 4: Output Growth instead of Consumption Growth, Band Pass Filtered data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
2-15 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-6.42E-07 [0]	7.62E-05 [0]	-2.48E-06 [8]	-7.00E-07 [0]	5.16E-06 [0]	-1.85E-07 [12]
Standard error	2.57E-05	0.006	1.15E-04	2.58E-05	2.12E-04	1.13E-04
Adjusted mean	-	-	-1.12E-06	-	-	4.92E-07
$E_t\pi_{t+1}$	0.310 [239]	0.671 [22]	0.132 [344]	0.326 [926]	1.472 [416]	0.894 [981]
Standard error	0.448	162.564	0.955	0.099	3.431	0.315
Adjusted mean	0.185	0.017	0.092	0.317	0.826	0.892
$E_tg_{y,t+1}$	0.020 [277]	-0.185 [40]	0.019 [237]	0.021 [408]	0.035 [163]	0.031 [460]
Standard error	0.017	16.569	0.027	0.012	0.582	0.027
Adjusted mean	0.011	0.012	0.012	0.013	0.041	0.024
E_tR_{t+1}	0.008 [147]	4.225 [27]	0.918 [524]	N/A	N/A	N/A
Standard error	0.475	88.304	1.002			
Adjusted mean	0.012	0.104	0.788			
$Mean$;						
R-square	0.169	<0	<0	0.153	<0	<0
Adjust R-square	0.142	<0	<0	0.136	<0	<0
Pr(F-statistic)	0.029 (887)	0.527 (162)	N/A	0.024 (891)	0.149 (599)	N/A
Pr(J-statistic)	N/A	N/A	0.050 {339}	N/A	0.352 {679}	0.058 {440}
Durbin-Watson	1.828 <850>	2.012 <974>	2.236 <991>	1.817 <783>	2.047 <996>	2.186 <990>
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.
· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table 5: Output Growth in a Standard Taylor Rule, Band Pass Filtered Data (2-15 years), 100 Years Simulated, 1000 Estimations Average.

$$R_t = \beta_0 + \beta_1 E_t\pi_{t+1} + \beta_2 E_t g_{y,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \tag{30}$$

This estimating equation corresponds to the misspecified representation of the Taylor condition with output growth plus further restrictions on the terms in productive time and the velocity of money; see equation (27). Equation (30) can be interpreted as a 'dynamic forward-looking Taylor rule' for $\beta_5 \neq 0$ or a 'static forward-looking Taylor rule' under the restriction $\beta_5 = 0$. Notably, the term in velocity growth is absent from this expression. This omission might be expected to have a bearing on the estimates because equations (28) and (29) produced a large number of statistically significant estimates for the velocity growth coefficient.

The results are again similar across the HP and band pass filters so only the $2-15$ band pass results are presented in Table 5.[22] The estimates are generally found to be poor in terms of the number of statistically significant estimates produced and in terms of mean R-square and adjusted R-square statistics. This is not surprising given that yet another source of misspecification has been added to the estimating equation.

The mean coefficient on inflation does not exceed unity for any of the three estimators considered. The results are also comparatively weak in terms of the frequency with which the null hypothesis of the F-statistic is rejected and in

[22]The instrument set now comprises of four lags of expected future inflation, four lags of expected future output growth, the second, third and fourth lags of the nominal interest rate and a constant term for the GMM estimator or just the shortest lag of each and a constant term for the exactly identified 2SLS estimator.

terms of the number of non-rejections of the null hypothesis of the Hansen J-test. The latter finding calls into question the validity of the instrument set used for the GMM estimator for equation (30).

The inflation coefficients are estimated surprisingly precisely under the restriction on β_5. However, these estimates differ quite substantially between estimating procedures for the $2-15$ filter; 0.317 (adjusted mean) for OLS compared to 0.892 for GMM (adjusted mean).

In short, imposing a 'conventional Taylor rule' restricts the true estimating equation to such an extent that the theoretical prediction that the coefficient on expected inflation exceeds unity is not verified. An estimated inflation coefficient of this magnitude might erroneously be interpreted to signify that the Taylor principle is violated but this result is simply a product of a misspecified estimating equation in the present context. Only if the model excluded physical capital and set velocity to one, by excluding exchange credit for example, would such an estimating equation be appropriate.

6 Low Frequency Results: Fisher Equation

The asset pricing equation (28) shows within the business cycle frequency the type of results we are familiar with from work in the Taylor literature, with the extensions of additional variables as discussed above. The simulation results that support our pricing equation are in the frequency range during which persistent shocks, such as our goods sector and bank sector productivity shocks and the money supply shock, continue to play out. In the lower frequency these shocks would tend to subside and the same equation (28) can be estimated in this frequency but it may not be expected to yield a Taylor type result. For example, at low frequencies the balanced growth path (BGP) relations may be expected to emerge within the pricing equation (28). In particular, it is conventional to think of the Fisher equation of nominal interest rates as tending to hold in the long run but with significant deviation from it over shorter horizons. Here we formalize this concept by showing that the results know typically as Taylor-type results that we find over the business cycle begin to fade into a Fisher interest relation over the lower frequencies.

Table 6 shows that, in the estimation of equation (28), as the Christiano and Fitzgerald (2003) filter is extended from an extended business cycle range of $2-15$ years out to $2-20$ years, the basic asset pricing equation is still estimated with significance for all of the variables and with the magnitude of the coefficients only falling somewhat, for example for the inflation coefficient by around 10%. Again, going to a lower frequency the third column shows that the asset pricing equation still finds significance for all variables, and now with the inflation coefficient dropping down further to 1.84.

Table 6: Estimation of Equation (26) with Extended Band Pass Filter Windows

Correctly Specified Model, Various Low Frequencies					
Filter window:	2-15 yrs.	2-20 yrs	2-25 yrs.	2-50 yrs.	2-75 yrs.
β_0	0.000 [22]	0.000 [24]	0.000 [30]	0.000 [31]	0.000 [44]
Standard error	0.000	0.000	0.000	0.000	0.000
Adj. mean	0.000	0.000	0.000	0.000	0.000
$E_t\overline{\pi}_{t+1}$	2.306 [1000]	2.068 [1000]	1.840 [1000]	1.097 [977]	0.813 [940]
Standard error	0.272	0.295	0.316	0.327	0.305
Adj. mean	2.306	2.068	1.840	1.089	0.798
$E_t\overline{g}_{c,t+1}$	0.302 [1000]	0.293 [1000]	0.280 [1000]	0.232 [995]	0.205 [975]
Standard error	0.025	0.031	0.036	0.048	0.053
Adj. mean	0.302	0.293	0.280	0.231	0.203
$E_t\overline{g}_{l,t+1}$	-0.359 [999]	-0.301 [991]	-0.241 [921]	-0.053 [334]	0.014 [246]
Standard error	0.085	0.093	0.098	0.112	0.108
Adj. mean	-0.359	-0.300	-0.235	-0.043	0.010
$E_t\overline{g}_{V,t+1}$	-0.269 [1000]	-0.276 [1000]	-0.270 [1000]	-0.230 [998]	-0.199 [983]
Standard error	0.031	0.033	0.035	0.046	0.052
Adj. mean	-0.269	-0.276	-0.270	-0.230	-0.198
$E_t\overline{R}_{t+1}$	-1.729 [1000]	-1.395 [987]	-1.114 [951]	-0.244 [367]	0.087 [293]
Standard error	0.322	0.376	0.421	0.454	0.428
Adj. mean	-1.729	-1.389	-1.100	-0.207	0.055
Mean:					
R-squared	0.782	0.764	0.750	0.687	0.664
Adj. R-squared	0.770	0.751	0.736	0.670	0.645
Pr(J-statistic)	0.315 {1000}	0.391 {1000}	0.436 {1000}	0.533 {1000}	0.526 {1000}
D-W	2.040 <881>	2.015 <860>	1.999 <874>	1.984 <872>	1.980 <892>
Pr(Q-statistic)	0.458 (913)	0.486 (940)	0.500 (949)	0.542 (962)	0.571 (970)
Adj. sample size	1000×96	1000×96	1000×96	1000×96	1000×96

However, as seen in the fourth column of Table 6, once the frequency extends to the $2-50$ year window, the asset equation looks very much like a Fisher equation of interest rate. The coefficient of the inflation rate is still highly significant and it takes on a value near to unity. And now there are only two other variables remaining significant. The consumption growth variable is highly significant with a coefficient value of 0.232, down from 0.30 in the extended business cycle range. The velocity variable also still highly significant with a coefficient of 0.23, down from 0.27 in the $2-15$ window. The growth rate in employment and the forward looking nominal rate variables are no longer significant.

The results of the $2-50$ year window suggest that this is where we see the Fisher equation of nominal interest rates holding. The interpretation is rather direct. The inflation coefficient is nearly one as in the Fisher equation, which is typically taken to imply in statistical testing that in the long run the nominal rate equals the inflation rate plus the real interest rate. Therefore the consumption growth rate in combination with the velocity growth takes on the meaning of the real interest rate. This is almost as in a conventional "Euler equation" of a real economy in which the real interest rate equals the consumption growth rate. So that part is perfectly consistent with the Fisher equation interpretation.

The addendum to the Fisher interpretation is that over time the velocity of money has trended up slightly but significantly. Benk et al. (2010) find for the 86 years from 1919 to 2004 that the average velocity growth rate was 1.26 a year. This means that velocity over the long term is not exactly stationary and so may impact upon, and appear within, the Fisher equation that emerges in this $2-50$ year window.

Consider Fisher's (1922) equation of exchange that $MV = Py$, where M is the money stock, V is velocity, P is the price level and y is the real income. First, the famous equation of exchange translates into the quantity theoretic concept that the money supply growth rate σ equals the inflation rate if V and y are constant. Given the well-known growth in output, which equals the growth in consumption along the *BGP*, we then get that the money supply growth equals the inflation rate plus the positive output growth rate. Velocity is typically taken as constant, both in Fisher's original work (when at the time velocity was quite stable) and in most related literature, with the strong exception of McCallum (1990). And indeed this stationary velocity is a *BGP* equilibrium of the model of Section 2 above. But now we will consider the Fisher equation would look at a low frequency that lies short of the *BGP* stationary equilibrium, in that some longer term modest secular trend upwards in velocity still affects our equation (24).

In this regard, consider that the Fisher equation is the *BGP* relation that $\overline{R} = \theta\overline{g} + \overline{\pi} + \rho$, so that the coefficient on inflation would be one. Then start with equation (24), and so consider that when Ω approaches 1, the coefficient on the expected inflation term is 1. In addition, with $\Omega = 1$ the coefficient on the forward interest rate term, $\Omega - 1$, is zero and so the forward interest rate term drops out. With employment growth in terms of hours per week becoming stationary, the corresponding term $\overline{g}_{l,t+1}$ would go to zero and drop out (it is -0.053 and insignificant in Table 6 below). This leaves the equation (24) as[23]

$$\overline{R}_t - \overline{R} = E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega_V E_t \overline{g}_{V,t+1}.$$

Comparable to equation (28) this gives the estimation model as

$$R_t = \beta_0 + \beta_1 E_t\pi_{t+1} + \beta_2 E_t g_{c,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t. \tag{31}$$

where the "near" *BGP* hypothesis that allows for an upwards velocity trend would be that $\beta_1 = 1$, $\beta_2 = \theta$, $\beta_3 = 0$, $\beta_4 \simeq -\Omega_V$ and $\beta_5 = 0$. This is consistent with how we interpret the Table 8 results in the $2-50$ year window. It is a Fisher -type equation in which the inflation effect on the nominal interest rate is slightly above one but then also slightly but significantly offset by velocity growth. We interpret this velocity effect as one that induces movement away from money towards exchange credit, and so reduces the impact of the given inflation rate on the nominal interest rate, just as we interpret the velocity effect in the $2-15$ year extended business cycle window.

Going beyond the $2-50$ year window, the inflation rate begins to lose significance as seen in the fifth column of Table 6, where its significance falls just below the 95% level. This suggests that the Fisher like relation holds in the very low frequency range of a band pass filter in the range of $2-50$ years and somewhat beyond that, but not as far as 75 years at which point the consumption and velocity growth terms stay significant but the inflation rate begins to lose significance.

The simulation results suggest that a Fisher-like equation emerges in the $2-50$ window, with only the inflation terms, consumption growth term and the velocity growth terms significant. The coefficient on inflation is near one. In terms of equation T2 this implies that one the naive equation fits well with only current values replacing expected values.

Corollary 5 *The theoretical coefficient on the inflation rate term equals 1 as in the Fisher equation when* $E_t\left(\overline{R}_{t+1}\right) = \overline{R}_t$.

Proof. When expectations fully adjust and $E_t\left(\overline{R}_{t+1}\right) = \overline{R}_t$ the solution equation (36), below, implies that

$$\begin{aligned} & \overline{R}_t + \lambda E_t\left(\overline{R}_{t+1}\right) = (1+\lambda)\overline{R}_t \\ = \quad & (1+\lambda) E_t\left(\overline{\pi}_{t+1}\right) + (1+\lambda)(0.5) E_t\left(\overline{g}_{c,t+1}\right) + (1+\lambda)\psi(1-\theta)\frac{l}{1-l} E_t \overline{g}_{l,t+1} \\ & -\Omega_V E_t \overline{g}_{V,t+1} + \rho. \end{aligned}$$

Dividing through by $(1+\lambda)$ gives back that

[23] Gathering together the constant terms on the right hand side, these being $\overline{R} - \overline{\pi} - \theta\overline{g}$, we reduce this express by using the *BGP* relation that $\overline{R} = \rho + \overline{\pi} + \theta\overline{g}$, $\overline{R} - \overline{\pi} - \theta\overline{g} = \rho$. But this assumes the Fisher equation holding without velocity entering. This addition means that $\overline{R} = \rho + \overline{\pi} + \theta\overline{g} - \Omega_V\overline{g}_V$ and the constant term would be $\overline{R} - \overline{\pi} - \theta\overline{g} = \rho - \Omega_V\overline{g}_V$.

$$\overline{R}_t = E_t\left(\overline{\pi}_{t+1}\right) + (0.5)\,E_t\left(\overline{g}_{c,t+1}\right) + \psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \frac{\Omega_V}{(1+\lambda)}E_t\overline{g}_{V,t+1} + \rho. \tag{32}$$

■

Combined with our medium term frequency results of the last table, this gives a Fisher equation equal to the expected inflation rate plus the real interest rate as given by the consumption growth $(0.5)\,E_t\left(\overline{g}_{c,t+1}\right)$, time preference factor, and an "inflation risk premium" due to persistent velocity change.

Indeed this leads to an understanding of why the inflation coefficient falls as the Medium Term Cycle window expands: as this horizon becomes longer, more of the expected future interest rate is from the current interest rate. This leaves some fraction of that future expected rate to get allocated to the current rate, and the rest to different expectations of the future rate. After the $2-25$ threshold that we identified during which our full dynamic specification of the interest rate remains significant for all of its composition terms, more and more of the expected future rate is allocated to the current rate, until the future rate becomes insignificant. As the window widens from $2-15$ all of the coefficients fall because part of the future nominal interest rate is allocated to the present R_t, brought to the left handside of the equation and then the equation is divided by the one plus that fractional allocation of E_tR_{t+1} to R_t. At first, this is closer to zero, and with wider windows closer to one, with insignificance of E_tR_{t+1} found at2 to50, along with a near unity inflation coefficient, in an internally consistent fashion seemingly.

In equation (28), at the $2-50$ window the $E_t\overline{g}_{l,t+1}$ term no longer is significant and so only the consumption growth and velocity growth terms remain along with the inflation term that has a near one coefficient. The coefficients of these terms along with $E_t\overline{g}_{l,t+1}$ before it loses significance, all trend down in a fashion consistent with dividing through the equation by 1 plus that part of E_tR_{t+1} that equals R_t, with that fraction increasing. Once to the $2-75$ window, the inflation term loses significance and so we restrict attention to the windows up to $2-50$.

The significance of velocity growth even at the far horizon is interesting. A rising money demand for a given money supply growth rate would induce less inflation. But given the inflation rate as well, rising money demand (or falling credit supply) would induce a higher real interest rate. This is consistent with the growth in the inverse of money demand, its velocity, causing a negative effect on the nominal interest rate. Apparently the growth of velocity represents an effect on the real interest rate, in particular driving it down. This occurs in the model, because given inflation, a rising velocity comes from increased credit productivity and greater credit issuance. This greater supply of credit apparently is the force behind velocity growth causing a significantly lower real interest rate even in the widest Medium Term cycle window.

7 Policy Evaluation

Here we set out what the interest rate equation (24) implies in terms policy analysis, as motivated by Lucas (1976). The solution of the interest rate equation will be used to analyze the effect of a policy change both in its correct form as well as when incorrectly assuming that the parameters of the forecasting equation remain unchanged. The policy forecast using the unchanged parameters gives a different answer than does the change in policy as rationally expected.

7.1 A Critique

Proposition 6 *The following solution is a special case of the general solution to equation (24).*

$$\overline{R}_t = (1+\lambda)\left[\overline{\pi}_t + \theta\overline{g}_{c,t} + \psi\left(1-\theta\right)\frac{l}{1-l}\overline{g}_{l,t} - \frac{\Omega_V}{(1+\lambda)}\overline{g}_{V,t} + \rho\right]. \tag{33}$$

Proof. Using the lag operator L, and forward operator L^{-1}, where $L^{-1}R_t = R_{t+1,}$ and letting $\lambda \equiv \Omega - 1$ equation can be written as

$$\overline{R}_t = \frac{(1+\lambda)\left[E_t\left(\overline{\pi}_{t+1}\right) + \theta E_t\left(\overline{g}_{c,t+1}\right) + \psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} - \frac{\Omega_V}{\Omega}E_t\overline{g}_{V,t+1} + \rho\right]}{\left[1+\lambda L^{-1}\right]}. \tag{34}$$

Appendix C shows how equation (34) can be written as equations (35)- (36):

$$
\begin{aligned}
\overline{R}_t &= (1+\lambda)\sum_{s=0}^{\infty}(-\lambda)^s \left[\begin{array}{c} E_{t+s}\left(\overline{\pi}_{t+s+1}\right)+\theta E_{t+s}\left(\overline{g}_{c,t+s+1}\right) \\ +\psi(1-\theta)\frac{l}{1-l}E_t\overline{g}_{l,t+s+1}-\frac{\Omega_V}{\Omega}E_t\overline{g}_{V,t+s+1}+\rho \end{array}\right]; \qquad (35)\\
\overline{R}_t &= (1+\lambda)\sum_{s=0}^{\infty}(-\lambda)^s \left[\begin{array}{c} \sum_{j=0}^{\infty}\Phi_{\pi}^{j}\overline{\pi}_{t-j}+\theta\sum_{j=0}^{\infty}\Phi_{g_c}^{j}\overline{g}_{c,t-j} \\ +\psi(1-\theta)\frac{l}{1-l}\sum_{j=0}^{\infty}\Phi_{g_l}^{j}\overline{g}_{l,t-j}-\frac{\Omega_V}{(1+\lambda)}\sum_{j=0}^{\infty}\Phi_{g_l}^{j}\overline{g}_{V,t-j}+\rho \end{array}\right]. \qquad (36)
\end{aligned}
$$

If we expand the equation (36) only to $s=j=0$ and ignore the rest of the terms, $j=1,2,3,...$, we get[24]

$$\overline{R}_t = (1+\lambda)\left[\overline{\pi}_t+\theta\overline{g}_{c,t}+\psi(1-\theta)\frac{l}{1-l}\overline{g}_{l,t}-\frac{\Omega_V}{(1+\lambda)}\overline{g}_{V,t}+\rho\right]. \qquad (37)$$

■

The resulting equation (37) has as a special case the familiar Taylor (1993) form of the interest rate equation, when employment and velocity growth are zero ($\overline{g}_{l,t}=\overline{g}_{V,t}=0$), and with consumption growth the "output gap" term.[25] The "Taylor principle" as taken to mean a coefficient >1 for the inflation term would be satisfied for all $R>0$. In contrast the full rational expectations solution would include all of the lagged terms that would enter according to equation (36) above.

7.2 Three Applications

1. Let the money supply growth rate permanently rise from Θ_1 to Θ_2.

   (a) Equation (35) view: The coefficient magnitudes would rise due to the increase in λ. The expected inflation rate will rise with certainty. Both of these factors will raise the impact of the inflation rate term on predictions of the policy change on nominal interest rates, and the increase in λ will increase the impact of the remaining terms on the nominal interest rate.

   (b) Equation (37) view with an understanding of the dependence of λ on structural/policy parameters: All of the coefficients in the equation will be estimated to be of a higher magnitude over frequencies up to a $2-25$ year Medium Term, or during a business cycle frequency of one's choice. But by not capturing the expected inflation increase in the inflation term, the equation would predict a lower nominal interest than would occur.

   (c) Taylor rule view of Equation (37) with fixed coefficients: The policy effect on nominal interest rates will be understated by the equation for the reasons of not changing λ and for not increasing expectations of the inflation rate. This will be a permanent understatement even as $t\to\infty$ since even once $\pi_t=E\pi_t$, λ will still be fixed below its true value for all t.

2. As a corollary, at time $t+T$, for $T\geq 0$ let the money supply growth rate be shocked upwards with a high autocorrelation on the shock. This could correspond for example to wartime spending financed by money supply growth, while leaving the *BGP* money supply growth rate constant.

   (a) Equation (35) view: While λ would stay constant, the expected future inflation rate would rise for an extended period of time after time $t+T$.

   (b) Equation (37) view with understanding of the dependence of λ on structural/policy parameters: The *BGP* money supply growth rate would not change and the λ would stay constant. The nominal interest rate would be underpredicted from time $t+T$ for a persisting period, since the higher expected inflation would not be built into the estimation until the inflation rate actually rose.

[24] Details of the proof are provided in the Appendix.

[25] We show above in equation (25) that the consumption growth term can be written equivalently in terms of output growth, with an additional investment growth term. This implies that the speed limit version of the Taylor equation is a special case of equation (37), which is a special case of equation (36).

(c) Taylor rule view of Equation (37) with fixed coefficients: same as in 2.b. since the coefficients would not change.

3. Bank crisis: a large drop in bank productivity A_F.

   (a) Equation (35) view: The decrease in A_F causes a increase in money demand and an expected decrease in its velocity. This causes λ to fall if this decrease in A_F is expected to be permanent. If not permanent, then it is one of the shocks of the economy, which include stochastic movement in A_F. Let it be viewed as permanent, so that λ falls. This in itself leverages down the effect of all of the variables in equation (35) on the nominal rate. The velocity growth coefficient Ω_V would also decline with an A_F decline; the net effect may be pressure downwards or upwards on the nominal rate from the velocity term. In addition, expected velocity growth would be negative, which by itself would add to upwards pressure on the current interest rate.

   (b) Equation (37) view with understanding of the dependence of λ on structural/policy parameters: The same as 3.a. except that the velocity growth would not factor in.

   (c) Taylor rule view of Equation (37) with fixed coefficients: This would not represent the correct "long run" specification of this equation for the business cycle frequency. The re-regulation would cause the λ to fall, but the this would not be captured so that the nominal interest rate would be predicted to be higher than by Equation (35). This upward bias in magnitude would carry across all parameters included in the estimated equation.

8 Discussion

The difference between equation (37) and related Taylor rule equations is that the coefficients of the Taylor rule are fixed while in equation (37) they depend on the *BGP* money supply growth rate. McCallum (2004) points out that Walters (1971) apparently first makes the argument that the solutions to such equations as (36) would have parameters that depend on the money supply process, while distributed lag coefficients with fixed parameters do not, and that this was an application of what became known as the Lucas (1976) critique more generally. In equations (35) to (37), the solution is consistent with Walters in that $\lambda = \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left(1+\overline{R}\right)\left[\gamma+\frac{m}{c}(1-\gamma)\right]} = \frac{(1-\gamma)A_F\left(\frac{\overline{R}\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}}}{\left(1+\overline{R}\right)\left(\gamma+\left[1-A_F\left(\frac{\overline{R}\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}}\right](1-\gamma)\right)}$ is an unambiguously positive function of the *BGP* money supply growth rate through the (log-utility) *BGP* nominal interest rate of $\overline{R} = \Theta^* + \rho$; so $\lambda = \lambda\left(\Theta^*\right)$. The coefficient rises as the *BGP* money supply growth rate rises.

Applying Equation (37) but without the velocity term or using a standard Taylor (1993) to explain pre 1981 and post 1981 US history of nominal interest rates leads to an underestimate of the nominal interest rate during the late 60s and 1970s and an overestimate of the nominal interest rate in the early 1980s. That is well known. Using Equation (35) would ameliorated that outcome. The expected future inflation rate jumped up during (the Vietnam War) period of the 60s and 70s; if this expected rate at any time t have been higher than the actual inflation rate at time t, the predicted nominal interest rate would have been higher and so closer to the actual nominal interest rate. Similarly during the Volcker regime that implemented the Congress mandated deflation in the early 1980s, if the expected deflation was greater at any time t than the actual deflation, then the predicted interest rate at time t would have been lower and so closer to the actual nominal interest rate. Equation (35) by the higher expected than actual inflation rate would be able to better explain interest rates in the pre 1981 period, and by a lower expected than actual inflation rate would be able to better explain interest rates in the post 1980 period.

Sørensen and Whitta-Jacobsen (2005, pp.502-505) present a related way in which the coefficients of the ‘rule’, under the assumption of constant money growth, relate to elasticities of money demand rather than the preferences of policy-makers. Alvarez et al. (2001), Schabert (2003) consider the link between money supply rules and interest rate rules.[26] Instead of an exogenous fraction of agents being able to use bonds as in Alvarez et al. (2001), here the consumer purchases goods with an endogenous fraction of bank-supplied intratemporal credit that avoids the inflation tax on exchange as found in Reynard (2004, 2006) and Benk et al. (2010). Schabert’s inflation coefficient in his interest rate equation is unity which we get in the special cash-only case (no credit, velocity fixed at one).

[26] Many others exist, for example Chowdhury and Schabert (2008).

Our paper might be viewed as trying to reconcile Canzoneri et al's. (2007) account of the empirical shortcomings of standard estimated Euler equations. As a corollary of this reconciliation we pursue Lucas (1976) and Lucas and Sargent's (1981) agenda of postulating "policy functions" with coefficients that depend explicitly upon the economy's underlying utility, technology, and policy parameters. The average growth rate of money supply on the *BGP* is a "structural" policy parameter that the consumer understands as part of the equilibrium conditions used to determine their behavior and which turn determines the *BGP* inflation rate which acts as the "target" inflation rate of the model.

9 Conclusion

The paper presents theoretically consistent estimation with simulated data of a 'Taylor rule' and a Fisher equation within the business cycle and Medium Term Cycle windows respectively. This paper shows how these well-known relations can be identified econometrically from the economy's asset pricing behavior over different data frequencies. Policy analysis with a standard Taylor rule can be viewed as a special case of the economy in which structural and policy coefficients do not change and naive expectations are the correct specification. The correct expectations solution to the interest rate equation correctly predicts known policy changes in the money supply growth rate, but the naive version incorrectly predicts the interest rate.

Standard Taylor rules in this Benk et al. (2010) world cannot be used as monetary policy rules since they incorrectly predict outcomes of policy change. A rule of a stationary mean growth rate of the money supply, with stochastic fluctuations due to fiscal finance, translates the economy into one in which a generalized Taylor rule holds. However this result is found true here only given correct expectations of the effects of the economy's set of stochastic shocks.

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A Alternative Equilibrium Interpretations of Asset Pricing

A.1 Backward Looking Taylor Condition

Consider two alternative representations of the Taylor condition; a backward-looking version and an alternative version which features credit. First, the Taylor condition can be reformulated to feature a lagged dependent variable on the right hand side instead of the lead dependent variable which appears in equation (24). This yields a similar expression written in terms of R_{t+1} instead of R_t:

$$\begin{aligned}\overline{R}_{t+1} - \overline{R} &= \frac{\Omega}{(\Omega-1)} E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \frac{\Omega\theta}{(\Omega-1)} E_t \left(\overline{g}_{c,t+1} - \overline{g}\right) \\ &\quad - \frac{\Omega\psi(\theta-1)\frac{l}{1-l}}{(\Omega-1)} E_t \overline{g}_{l,t+1} - \frac{\Omega_V}{(\Omega-1)} E_t \overline{g}_{V,t} - \frac{1}{(\Omega-1)} \left(\overline{R}_t - \overline{R}\right). \end{aligned} \tag{38}$$

While equation (38) compares better to interest rate rules which feature a lagged dependent variable on the right hand side as an ‘interest rate smoothing’ term, the lead nominal interest rate is now the dependent variable. Such an expression is more akin to a forecasting equation for the nominal interest rate than an interest rate rule. Such a transformation also raises the fundamental issue discussed by McCallum (2010). He argues that the equilibrium conditions of a structural model stipulate whether any given difference equation is forward-looking ("expectational") or backward-looking ("inertial") and that the researcher is not free to alter the direction of causality implied by the model as is convenient. The forward looking representation of the Taylor condition (24) is the long accepted rational expectations version; for example, Lucas (1980) suggests that the forward looking "filters" suit models which feature an optimizing consumer. In fact, we would argue that the timing of the cash-in-advance economy is such that our forward-looking rule in equation (24) is the correct model, while equation (38) is consistent with the alternative "cash-when-I'm-done" timing which we do not employ (see Carlstrom and Fuerst, 2001).

A.2 Credit Interpretation of the Taylor Condition

Christiano et al. (2010) have considered how the growth rate of credit might be included as part of a Taylor rule so that "allowing an independent role for credit growth (beyond its role in constructing the inflation forecast) would reduce the volatility of output and asset prices." The term in velocity growth can be rewritten as the growth rate of credit in the following way: Since $V_t = \frac{c_t}{m_t} = \frac{1}{1-\left(1-\frac{m_t}{c_t}\right)}$, then $V\widehat{V}_t = \left(1-\frac{m}{c}\right)\widehat{\left(1-\frac{m_t}{c_t}\right)}$ and $\overline{g}_{V,t} = \frac{m}{c}\left(1-\frac{m}{c}\right)\overline{g}_{\left(1-\frac{m}{c}\right),t}$ where $\overline{g}_{\left(1-\frac{m}{c}\right),t}$ is the growth rate of normalized credit. The Taylor condition is now rewritten as:

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t \left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi(1-\theta)\frac{l}{1-l} E_t \overline{g}_{l,t+1} \\ &\quad - \Omega_{\left(1-\frac{m}{c}\right)} E_t \overline{g}_{\left(1-\frac{m}{c}\right),t+1} - \Omega_R E_t \left(\overline{R}_{t+1} - \overline{R}\right). \end{aligned} \tag{39}$$

The credit coefficient can be derived as $\Omega_{\left(1-\frac{m}{c}\right)} \equiv \frac{(R-1)(1-\gamma)\left(\frac{m_t}{c_t}\right)^2}{(1-\gamma)^2} \frac{(1-\gamma)\left(1-\frac{m_t}{c_t}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m_t}{c_t}\right)\right]} \geq 0$. A positive expected credit growth rate decreases the current net nominal interest rate $\overline{R}_t$. With velocity set at one as in a standard cash-in-advance economy, neither credit nor velocity would enter the Taylor condition since the credit service does not exist and velocity does not vary over time.

B Results with Alternate Filters for Estimation

Tables B-1 and B-2 show the similar business cycle window results of estimations of Equation (28) using the HP filter (Ravn and Uhlig (2002) Smoothing Parameter) and a 3 to 8 Christiano and Fitzgerald (2004) band pass filter, instead of the 2 to 15 window presented in the main body of the paper.

HP filtered data,	Unrestricted			Assumed $\beta_5 = 0$		
where HP $\lambda = 6.25$	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-9.68E-07 [0]	-2.04E-07 [0]	8.09E-07 [17]	-7.60E-07 [0]	-4.78E-07 [0]	-7.57E-08 [8]
Standard error	2.87E-05	2.15E-05	3.15E-05	2.39E-05	1.67E-05	4.29E-05
Adjusted mean	-	-	-4.27E-08	-	-	2.59E-07
$E_t\pi_{t+1}$	2.019 [1000]	2.309 [691]	2.299 [1000]	0.315 [830]	0.757 [397]	0.621 [925]
Standard error	0.248	1.488	0.268	0.126	0.856	0.265
Adjusted mean	2.019	1.800	2.299	0.293	0.475	0.614
$E_t g_{c,t+1}$	0.251 [1000]	0.336 [959]	0.293 [1000]	0.172 [1000]	0.313 [989]	0.231 [1000]
Standard error	0.024	0.096	0.020	0.020	0.048	0.025
Adjusted mean	0.251	0.324	0.293	0.172	0.311	0.231
$E_t g_{l,t+1}$	-0.243 [890]	-0.536 [774]	-0.374 [997]	-0.281 [864]	-0.530 [774]	-0.427 [996]
Standard error	0.094	0.321	0.079	0.100	0.231	0.111
Adjusted mean	-0.236	-0.448	-0.374	-0.265	-0.453	-0.427
$E_t g_{V,t+1}$	-0.137 [990]	-0.267 [800]	-0.212 [1000]	-0.098 [889]	-0.317 [888]	-0.190 [992]
Standard error	0.031	0.228	0.033	0.036	0.109	0.056
Adjusted mean	-0.137	-0.229	-0.212	-0.093	-0.293	-0.190
$E_t R_{t+1}$	-1.819 [1000]	-2.338 [646]	-2.005 [1000]	N/A	N/A	N/A
Standard error	0.221	2.282	0.277			
Adjusted mean	-1.819	-1.692	-2.005			
$Mean$;						
R-square	0.789	<0	0.796	0.544	<0	0.482
Adjusted R-square	0.778	<0	0.785	0.525	<0	0.459
Pr(F-statistic)	2.35E-15 (1000)	0.015 (974)	N/A	3.93E-09 (1000)	0.003 (992)	N/A
Pr(J-statistic)	N/A	N/A	0.258 {1000}	N/A	0.159 {482}	0.269 {1000}
Durbin-Watson	1.474 <151>	2.243 <1000>	2.194 <970>	1.732 <419>	2.145 <999>	2.047 <882>
Sample size (1000x)	99	98	96	99	98	96

Notes:
· 'Standard error' measures the variation in the coefficient estimates.
· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.
· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).
· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).
· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.
· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and <> the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table B-1: Taylor Condition Estimation, HP Filtered Data, Ravn and Uhlig (2002) Smoothing Parameter, 100 Years Simulated, 1000 Estimations Average.

BP Filter,	Unrestricted			Assumed $\beta_5 = 0$		
3-8 Window	OLS	2SLS	GMM	OLS	2SLS	GMM
β_0	-7.57E-07 [0]	6.54E-06 [0]	-7.10E-07 [3]	7.73E-07 [0]	-2.86E-06 [0]	-1.51E-06 [1]
Standard error	1.65E-05	3.24E-04	2.09E-05	1.45E-05	4.37E-05	2.68E-05
Adjusted mean	-	-	-1.83E-07	-	-	-9.81E-08
$E_t \pi_{t+1}$	2.166 [998]	2.484 [724]	2.423 [1000]	0.633 [969]	2.417 [965]	0.682 [974]
Standard error	0.391	32.906	0.298	0.195	1.125	0.222
Adjusted mean	2.166	2.141	2.423	0.628	2.291	0.679
$E_t g_{c,t+1}$	0.283 [1000]	0.304 [623]	0.314 [1000]	0.155 [1000]	0.175 [834]	0.168 [1000]
Standard error	0.043	7.324	0.027	0.029	0.074	0.030
Adjusted mean	0.283	0.231	0.314	0.155	0.160	0.168
$E_t g_{l,t+1}$	-0.237 [827]	-0.573 [430]	-0.312 [982]	-0.222 [685]	-0.595 [666]	-0.268 [870]
Standard error	0.131	10.199	0.099	0.133	0.361	0.135
Adjusted mean	-0.229	-0.193	-0.312	-0.195	-0.455	-0.259
$E_t g_{V,t+1}$	-0.152 [982]	-0.453 [351]	-0.174 [998]	-0.174 [976]	-0.604 [973]	-0.194 [984]
Standard error	0.043	15.068	0.039	0.052	0.252	0.057
Adjusted mean	-0.151	-0.128	-0.174	-0.173	-0.578	-0.193
$E_t R_{t+1}$	-2.026 [994]	-1.532 [424]	-2.289 [1000]			
Standard error	0.435	116.012	0.339			
Adjusted mean	-2.024	-1.211	-2.289			
$Mean$;						
R-square	0.789	<0	0.842	0.576	<0	0.590
Adjusted R-square	0.778	<0	0.833	0.558	<0	0.572
Pr(F-statistic)	4.75E-10 (1000)	0.077 (874)	N/A	2.08E-07 (1000)	0.005 (981)	N/A
Pr(J-statistic)	N/A	N/A	0.213 {1000}	N/A	0.344 {848}	0.249 {1000}
Durbin-Watson	1.568 $<54>$	1.653 $<330>$	1.517 $<49>$	1.728 $<333>$	1.728 $<378>$	1.715 $<306>$
Sample size (1000x)	99	98	96	99	98	96

Notes:

· 'Standard error' measures the variation in the coefficient estimates.

· 'Adjusted mean' assigns a value of zero to non statistically significant estimates.

· F-statistic: null hypothesis of no joint significance of the independent variables (not available under GMM).

· J-statistic: null hypothesis that the instrument set is valid (only available if there are over-identifying restrictions).

· Durbin-Watson statistic: null hypothesis that successive error terms are serially uncorrelated against an AR(1) alternative.

· [ ] reports the number of statistically significant coefficient estimates, () the number of F-statistic rejections, {} the number of J-statistic non-rejections and $<>$ the number of times the D-W statistic exceeds its upper critical value (all at the 5% level of significance).

Table B-2: Taylor Condition Estimation, Band Pass Filtered Data (3-8 years), 100 Years Simulated, 1000 Estimations Average.

C Proof of Proposition 5

The solution to each $\overline{\pi}_t$, $\overline{g}_{c,t}$, $\overline{g}_{l,t}$, and $\overline{g}_{V,t}$ for all t is a nonlinear function of the state variables consisting of the three shocks, u_t, v_t, and z_t, and the capital ratio k_t/h_t, which are all given at time period t, end of period. Shocks are given as

$$\begin{aligned}
u_t &= \rho_u u_{t-1} + e_{ut} = \sum_{j=0}^{t} \rho_u^{t-j} e_{u,j+1}; \\
v_t &= \rho_v v_{t-1} + e_{vt} = \sum_{j=0}^{t} \rho_v^{t-j} e_{v,j+1}; \\
z_t &= \rho_z z_{t-1} + e_{zt} = \sum_{j=0}^{t-1} \rho_z^{t-j} e_{z,j+1}.
\end{aligned}$$

Using function forms for our solution, they can be written as $\overline{\pi}_t^* = \Pi(u_t, v_t, z_t; k_t/h_t)$, $\overline{g}_{c,t}^* = C(u_t, v_t, z_t; k_t/h_t)$, $\overline{g}_{l,t}^* = L(u_t, v_t, z_t; k_t/h_t)$, $\overline{g}_{V,t}^* = V(u_t, v_t, z_t; k_t/h_t)$. Taking a linear approximation, write these solutions with a constant term as $\overline{\pi}_t^* = \Pi + \Pi_u u_t + \Pi_v v_t + \Pi_z z_t + \Pi_{k/h}(k_t/h_t)$; $\overline{g}_{c,t}^* = C + C_u u_t + C_v v_t + C_z z_t + C_{k/h}(k_t/h_t)$; $\overline{g}_{l,t}^* = L + L_u u_t + L_v v_t + L_z z_t + L_{k/h}(k_t/h_t)$; $\overline{g}_{V,t}^* = V + V_u u_t + V_v v_t + V_z z_t + V_{k/h}(k_t/h_t)$.

As McCallum (2004) exposits for an economy with autocorrelated shocks, the Lucas (1972) solution involves "only the shocks and variables recognized to be relevant to the current state of the system, i.e., the relevant state variables." Using an autocorrelated shock similar to the ones in our model and restated above, the solutions for each variable is a function of the state variables. This is how the model in Benk et al. (2005, 2008, 2010) is solved.

If the economy starts at the *BGP* equilibrium, where that is time 0, then before the shocks occur, $\pi_0 = \Pi + \Pi_{k/h}(k_0/h_0) = \pi$, the *BGP* equilibrium value of the inflation rate.[27] The solution form for each of the variables puts them in a form similar to one as a function of a permanent (eg. $\Pi_{k/h}(k_t/h_t)$), a temporary (eg. $\Pi_u u_t + \Pi_v v_t + \Pi_z z_t$) and a constant term (eg. Π) as in Muth (1961) and Lucas (1976) in describing Friedman's (1957) income hypothesis. Taking the solution for R_t by converting the forward operator, and using Muth (1961), Lucas (1976) and Sargent (1973) that gives that expected future inflation $E_{t+s}(\overline{\pi}_{t+s+1})$ for all $s = 0, 1, 2, 3, ...$ to be the distributed lag $\sum_{j=0}^{\infty} \Phi_\pi^j \overline{\pi}_{t-j}$, we can likewise characterize the other expected future variables and present the solution as in the proof.

D Consumer's First-Order Conditions

$$\max_{c_t, x_t, l_{Gt}, l_{Qt}, s_{Ft}, q_t, d_t, k_{t+1}, h_{t+1}, M_{t+1}} \mathscr{V}(k_0, h_0, M_0; z_0, u_0, v_0) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t x_t^\Psi)^{1-\theta}}{1-\theta} \tag{40}$$

subject to

$$\begin{aligned}
\lambda &: \quad w_t(l_{Gt} + l_{Ft})h_t + r_t s_{Gt} k_t + R_{Ft} d_t + \frac{T_t}{P_t} && (41) \\
&\geq \quad \frac{P_{Ft}}{P_t} q_t + c_t + k_{t+1} - (1-\delta_K)k_t + \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} \\
\mu &: \quad \frac{M_t}{P_t} + \frac{T_t}{P_t} + q_t \geq c_t && (42) \\
\varepsilon &: \quad c_t = d_t && (43) \\
\psi &: \quad h_{t+1} = (1-\delta_H)h_t + A_H \left[(1 - l_{Gt} - l_{Ft} - x_t)h_t\right]^\eta \left[(1-s_{Gt})k_t\right]^{1-\eta}. && (44)
\end{aligned}$$

[27] Lucas and Prescott (1971) show that the equilibrium state variable "settles down" to the average which is here the *BGP* equilibrium value when the economy faces such autocorrelated shocks. So we can take the time t value of $\Pi + \Pi_{k/h}(k_t/h_t)$ to be close to π when the economy is close to its *BGP* equilibrium at time t. Similarly, $C + C_{k/h}(k_t/h_t)$ is close to the *BGP* value of $\overline{g}_c$, and $L + L_{k/h}(k_t/h_t)$ and $V + V_{k/h}(k_t/h_t)$ close to $\overline{g}_l$ and to $\overline{g}_V$.

$$
\begin{aligned}
0 &= (c_t x_t^{\Psi})^{-\theta} x_t^{\Psi} - \lambda_t - \mu_t + \varepsilon_t, \\
0 &= \Psi c_t^{1-\theta} x_t^{\Psi(1-\theta)-1} - \psi_t \eta A_H h_t \left(l_{Ht} h_t\right)^{\eta-1} \left(s_{Ht} k_t\right)^{1-\eta}, \\
0 &= \lambda_t w_t h_t - \psi_t \eta A_H h_t \left(l_{Ht} h_t\right)^{\eta-1} \left(s_{Ht} k_t\right)^{1-\eta}, \\
0 &= \lambda_t w_t h_t - \psi_t \eta A_H h_t \left(l_{Ht} h_t\right)^{\eta-1} \left(s_{Ht} k_t\right)^{1-\eta}, \\
0 &= \lambda_t r_t k_t - \psi_t (1-\eta) A_H k_t \left(l_{Ht} h_t\right)^{\eta} \left(s_{Ht} k_t\right)^{-\eta}, \\
0 &= -\lambda_t \left(\frac{P_{Ft}}{P_t}\right) + \mu_t, \\
0 &= \lambda_t R_{Ft} - \varepsilon_t, \\
0 &= -\lambda_t + \beta E_t \left\{\lambda_{t+1} \left[1 - \delta_K + r_{t+1} s_{G,t+1}\right]\right\} \\
&\quad + \beta E_t \left\{\psi_{t+1} (1-\eta) A_H s_{Ht+1} \left(l_{Ht+1} h_{t+1}\right)^{\eta} \left(s_{Ht+1} k_{t+1}\right)^{-\eta}\right\}, \\
0 &= -\psi_t + \beta E_t \left\{\lambda_{t+1} w_{t+1} \left(l_{Gt+1} + l_{Ft+1}\right)\right\} \\
&\quad + \beta E_t \left\{\psi_{t+1} \left[1 - \delta_H + \eta A_H l_{Ht+1} \left(l_{Ht+1} h_{t+1}\right)^{\eta-1} \left(s_{Ht+1} k_{t+1}\right)^{1-\eta}\right]\right\}, \\
0 &= -\frac{\lambda_t}{P_t} + \beta E_t \left\{\frac{\lambda_{t+1} + \mu_{t+1}}{P_{t+1}}\right\}.
\end{aligned}
$$