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Supply-Side Economics with AS-AD in Ramsey Dynamic General Equilibrium*

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Abstract

The Ramsey dynamic general equilibrium (RDGE) model has been applied broadly within mainstream macroeconomic analysis. While the labor market of the RDGE model has long been developed, any consensus on the goods market has remained elusive. This has made supply-side policy analysis within it difficult since it is founded upon the premise of aggregate supply (AS) featuring more prominently than aggregate demand (AD). Specifying a relative price of output that makes the goods market consistent with the recursive structure of the RDGE paradigm, the paper then applies AS-AD quantitatively to study productivity increases and income tax rate decreases that have been a centerpiece in supply-side economics. The paper contributes how both a productivity increase and a capital income tax rate decrease cause a net shift out of AS relative to AD that lowers the relative price of output. It shows how productivity increases and tax rate reductions quantitatively increase macroeconomic variables. The increase in economic activity remains proportional to the percentage increase in productivity, giving rise to our introduction of the concept of a productivity multiplier. Tax rate reductions cause increased economic activity at a decreasing rate as the level of the tax rate decreases. The paper shows the sense in which capital income tax rate reductions quantitatively have larger magnitude effects on output, consumption, investment, and capital wealth, while labor income tax rate reductions have larger magnitude effects on employment. Tax revenue implications are also presented with the first Laffer curves linked to the RDGE AS-AD analysis. Limitations, extensions and policy applications are suggested.

JEL Classification: O40, E61, E13, B41

Keywords: Supply-Side Economics, Aggregate Supply and Aggregate Demand, Productivity Increase, Productivity Multiplier, Tax Rate Decrease, Laffer Curve.

"You cannot crush a concept." Martin Booth, *The American*

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1 Introduction

Supply-side economics remains a widespread global concept popular in policy analysis for productivity and tax reform. Supply-side economics emphasizes how productivity increases affect long run output, investment and employment and how permanent tax rate reductions likewise increase these macroeconomic variables. Quantifying supply-side economic effects through the use of aggregate supply and demand analysis remains stymied by the lack of a fully price-theoretic, microfounded, optimization model of aggregate output supply and demand. Instead a mix of Keynesian, New-Keynesian, neoclassical and real business cycle theory provides analysis of temporary shocks to widely different definitions of aggregate supply and demand without consensus on the methodology and with a focus on aggregate fluctuations. This leaves a vacuum for analyzing permanent effects stressed traditionally by supply-side economics through aggregate supply and demand for output. To provide a paradigm filling this void, the analysis is here derived through the modern rational expectations approach of a single set of equilibrium conditions for the representative agent optimization problem without added "blocks" of equilibrium conditions.

First, this paper contributes a framework for quantifying supply-side analysis that modifies qualitative textbook treatments critically by turning the nominal price of output into its relative price that depends on the stationary equilibrium capital stock. It derives this as a closed-form solution through a single fully microfounded optimization problem that underlies most of modern macroeconomics. Second, its most important contribution is the first quantitative application of this internally consistent aggregate output supply and demand (AS-AD) to the analysis of supply-side economics. It quantifies how productivity increases and tax rate reductions compare, includes closed form solutions of the Laffer curve for each the labor income tax rate and the capital income tax rate, and qualifies the results by discussing its limitations and suggesting extensions to address these.

Using the canonical Ramsey (1928) dynamic general equilibrium model (RDGE) that forms the foundation of both the New-Keynesian and neoclassical real business cycle theories of aggregate output, it leaves no room for discounting its value-added on the grounds that the model lacks consensus as the foundation of modern dynamic analysis. There is no lack of consensus that the RDGE model is the microfounded basis of both main modern DSGE approaches, as well as others such as New-Monetarism and a realm of related approaches. This allows us to contribute a new set of results that goes well beyond existing quantification of traditional supply-side analysis.

The paper details how productivity increases and capital income tax rate decreases both cause aggregate supply to shift out by more than aggregate demand. The capital income tax rate reductions more closely mimic productivity increases as compared to labor income tax rate reductions because the latter cause supply and demand to shift out equally. Capital tax rate decreases also create higher income, investment, wealth and employment and a lesser decrease in government revenue than do proportionate labor income tax rate decreases. We identify Laffer curves in the results for both the labor income and capital income tax rates. Once revenue starts decreasing from tax rate reductions, we find smaller increases in output and related variables the lower the tax rate goes. The paper also shows that percentage increases in productivity have a constant proportionate expansionary effect across effected variables effected, which we term the "productivity multiplier."

Using a calibrated model, these results are robust to non-extreme values of parameters. They pertain to the stationary state. There is no short run AS-AD except for transitional dynamics that we do not report. Rather these represent what we think of as the long run even as the aggregate supply of output is upward-sloping in the stationary state, except for an extreme Cobb-Douglas parameterization. The growth effects of tax rate reductions and whether tax revenue reductions can be made up through higher consequent growth are additional themes of supply-side economics that are beyond the purview of the RDGE model since the growth rate is exogenously zero, or alternatively a positive exogenous Solow rate of growth. The paper also leaves out optimal tax evasion, an important theme in tax reform. Therefore the paper contributes analysis of the first two parts of supply-side economics through the RDGE AS-AD framework, of productivity and tax reform effects, and leaves for further work how supply-side tax rate reduction policy can increase growth, cause less incentive for tax evasion, and increase the size of the tax base.

The next Section 2 provides a literature review. Section 3 sets out the RDGE model, Section 4 the derivation of AS-AD analysis, and Section 5 the application to a productivity increase. Section 6 extends the RDGE model with income taxes and shows the effects of tax rate reduction. Section 7 discusses limitations of the results and possible extensions and Section 8 concludes.

2 Literature Review

Supply-side economics arose in the late 1970's with the prominent "Supply Side" contribution of Klein (1978) followed by "Supply-Side Economics" of Laffer (1981). Combined with the 1981 and 1986 tax rate reductions under President Reagan in the US, this unleashed a steady stream of 1980s supply-side economics research about President Kennedy's tax rate reductions (Heller , 1981), Reagan's reductions (Cutler and Summers , 1988; Marshall and Arestis , 1989), general ideology (Weintraub , 1981; Helliwell , 1986; Rousseas , 1983), and graphical representations Shapiro (1981). The related AS-AD analysis struggled to formalize supply-side economics, for example in the Bruno (1985) analysis of unemployment. Overviewed by Roberts (2003), he writes that

> "Supply-side economics is a major innovation in economics. It says that fiscal policy works by changing relative prices and shifting the aggregate supply curve, not by raising or lowering disposable income and shifting the aggregate demand curve. Supply-side economics reconciled micro- and macroeconomics by making relative-price analysis the basis for macroconclusions ... Supply-side economics presented a fundamental challenge to Keynesian demand management...Keynesian economists objected to the fiscal emphasis on relative price effects....Supply side economics brought the insight that marginal tax rates enter directly into the cost of capital. ... The relative-price effects will expand the economy ... By the time of Ronald Reagan's election as president, there was bipartisan support for a supply-side change in the policy mix. Inflation would be restrained by monetary policy, and output would be expanded by lowering the after-tax cost of labor and capital (pp. 393-395)."

A revival in the 1990s was led by the Lucas (1990) dynamic capital income tax reduction analysis. Tax rate reductions and their incentive effects on macroeconomic variables were studied in a wide variety of ways, for example by Perkins (1992), Creedy and MacDonald (1993), Dunn Jr. and Cordes (1994), Kyer and Maggs (1994), and Campbell (1994). This included microfounded supply-side neoclassical applications of relative price effects to related issues including inequality in Topel (1997). Meanwhile confusion on the AS-AD model continued. Bhaduri et al. (1994) described logical inconsistencies in the AS-AD traditional framework and Colander (1995) argued that internal inconsistencies exist in the most widely used analysis.

This led to various revisions of AS-AD analysis such as with Docherty and Tse (2009) and Benigno (2015), applications to tax reform analysis in James and Edwards (2008), secular stagnation in (Gordon , 2015), and fiscal rule scenarios in Iacono (2017). Lipsey (2010) reviews a large set of AS-AD diagrams; Saez and Michaillat (2013) uses AS-AD "with product market tightness acting as a price;" Mora (2018) applies a version to model the Venezuelan economy; Reifschneider et al. (2015) explore whether aggregate supply is endogenous with aggregate demand New-Keynesian models. Dutt (2006) posits that aggregate demand is not present in Neoclassical models but is in Keynesian models.

A recent focus emphasizes shocks instead of permanent effects for supply-side issues. This can be studied empirically in Estrella (1997), Bekaert et al. (2020), Stuart (2022), Calvert-Jump and Kohler (2022), Bi (2023) and Wang et al. (2023). The theoretical shock approach includes New Keynesian analysis of a log-linearized Keynesian cross (Bilbiie , 2020), using definitions of aggregate demand and supply that are compared to Samuelson's "original" one even though these are in terms of shock deviations from the steady state rather than in levels. This genre of analysis based on mark-up shocks that drive inflation has been applied to secular stagnation (Laubach and Williams , 2003; Holston et al. , 2015), fiscal multipliers (Bilbiie and Melitz , 2022) and aggregate demand and supply shocks (Baqaee and Farhi , 2021). Consistent with this approach, Mankiw (2018) uses a nominal price for an AS-AD analysis with oil shocks to describe the 1970s stagflation and cost-push inflation. Taylor (2000) and Taylor and Weerapana (2022) define the price for his AD curve by the percentage change in the nominal price level, the inflation rate, omit an AS curve, and promote the Taylor (1993) rule without money in the analysis.

Historically, there is a bifurcation in the development of what the price is for aggregate demand that helps explains the modern lack of consensus on a microfounded AS-AD analysis of supply-side economics. The quantity theory of Fisher (1911) posits the nominal price level as being determined mainly by the money supply level. Keynes (1923) advocates the quantity theory as a way to stabilize prices by understanding that changes in velocity is a behavioral factor that policymakers could anticipate. With the onset of the Great Depression, Keynes (1930) explicitly puts aside the quantity theory and the use of money to define the nominal price level and instead advocates a new theory of the aggregate price level based on his Professor Marshal's microeconomic theory of the firm, combined with national income accounts. Keynes (1930) argues that the aggregate nominal price per unit of output equals average cost plus profit and that profit equals investment minus savings as normalized by output (with the latter part being incorrect; see Gillman (2002)). The New Keynesian model exactly picks up this theme by making the aggregate price level equal to average cost plus profit, omits the money supply, and defines inflation as the percentage change in this price. The New Keynesian "aggregate supply" of output is then defined in the 3-equation model as dependent upon a permanent Phillips curve that lets output rise with the inflation rate, making the inflation rate the key "price" of the model. Since inflation only rises due to the markup shock and otherwise is zero in the 3-equation model, the model's nomenclature for aggregate demand and supply is based on shocks that cause inflation, leading up to a neoclassical reinterpretation (without capital) of Keynesian supply shocks in (Guerrieri et al. , 2022).

Neoclassical development saw the Hicks ian (1946) demand curve with utility held constant. Applied to a static aggregate economy without capital accumulation, this is based on a relative price derived from the marginal rate of substitution between labor and goods at the tangency point. The relative price is the amount of time required per good. That may sound like Marxian doctrine based in the classical labor theory of value, but in that theory supply alone determines output. With the onset of the neoclassical revolution of adding demand into the determination of relative prices in Marshall (1920), the Hicks ian (1946) demand in aggregate theory gives a relative price based on time given up per good as determined by the equilibrium of supply and demand. For the alternative Marshallian(1920) demand curve, movement along the demand as the relative price changes allows the utility level to change as well, leading to more price elastic demand as Friedman (1976) describes. The problem was to extend the Hicks-Marshall demand to aggregate analysis with dynamic optimization and capital accumulation in a fully microfounded framework. Ramsey (1928) provided that model that today forms the basis of modern dynamic macro and that has a relative price of output contained within it, including dependence on the equilibrium capital stock.

Supply-side economics then faced the problem of deriving AS-AD consistent with the RDGE model. For example, Mitchell (1949) argues that no such dynamic theory of value in general equilibrium exists that specifies aggregate supply and aggregate demand based on the equilibrium relative price of output. Since Keynes (1930), aggregate demand and supply has been dominated by a changing set of definitions without a simple microeconomic-based relative price that results from optimization of a dynamic general equilibrium model, being "Stories that we Tell" as Colander (1995) puts it.

The IS-LM analysis of Hicks (1937) has long been viewed as an alternative aggregate supply and demand analysis, with IS as AD, LM as AS, and the interest rate (real and nominal) as the relative price that is not derived from optimization. Klein (1978) suggested original sins in non-microfounded AS-AD analysis: "Keynes probably confused the issues by making labor supply dependent on the nominal wage rate, assuming the existence of money illusion, and by not treating the stock of capital as an explicit variable." The lack of internal consistency needed for supply-side economic analysis was pointed out by Helliwell (1986): "Nor does the strong empirical support for the 'old' supply-side economics mean that the incentive effects emphasized in the 'new' supply-side economics are without content. Rather, as emphasized by Feldstein (1986), their relative importance can only be assessed properly when they are integrated into a complete macro-economic framework that permits factor supplies and output to be determined in a mutually consistent manner." A related perspective on the need for internally consistent models to analysis productivity and tax rate changes can be viewed through Samuelson's (1947, p. 258) "correspondence principle". This states that "the problem of stability of equilibrium is intimately tied up with the problem of deriving fruitful theorems in comparative statics."

Such comparative statics cannot be defined for shock deviations from equilibrium, which is why impulse responses are popular, but rather are the change from one stationary equilibrium to another even if the model is dynamic.[1] Keynes 's student Ramsey (1928)

[1] Boulding (1955, p. 486) writes that comparative static analysis of a fully dynamic system is valuable and exactly the second of two principles that Samuelson's (1947) *Foundations of Economic Analysis* advocated: "The first is the clear distinction between comparative statics and process analysis. The second is the demonstration that the stability of an equilibrium system can only be established if the equilibrium system itself can be expressed as a limiting position of a dynamic model." He describes how solutions to comparative static equilibria within dynamic systems are derived without dynamic optimization: "The 'solution' is that set (or sets) of values of the variables which is consistent with a given set of relationships and parameters as expressed in the equations. In comparative statics we compare one 'solution' derived from one set of parameters with another solution derived from a different

states in his seminal work that his RDGE model is merely a formalization of what Keynes relayed to him. Ackley (1969, p. 436, Figure 10-2) graphs AS-AD with the price of aggregate output as the dollar price per good (P) divided by the dollar price per time defined as the nominal wage rate (W). We show that this relative price, the real amount of time required to be given up per real unit of the aggregate output, is found within the RDGE model. With this we show that the Harberger (1998) point, about an increase in productivity lowering the relative price of output, is a supply-side economics result from aggregate supply shifting out by more than aggregate demand. Building on Ackley (1969) to achieve Mitchell's (1949) goal of a dynamic theory of value based on the stationary equilibrium, the paper shows that such RDGE supply-side economic analysis with AS-AD depends upon the stationary level of the capital stock.

3 Representative Agent Economy

With $A \in R_{++}$ being the constant productivity parameter, $\phi \in [0,1]$, y_t denoting goods output at time t, l_t denoting labor input, and k_t denoting capital stock input, let the technology of the goods producer be Cobb-Douglas such that

$$y_t = A\left(l_t\right)^{\phi}\left(k_t\right)^{1-\phi}. \tag{1}$$

Competitive profit maximization implies that the wage rate, denoted by w_t, and the real interest rate, denoted by r_t, are given in equilibrium by

$$w_t = \phi A\left(\frac{l_t}{k_t}\right)^{\phi-1}, \tag{2}$$

$$r_t = (1-\phi)A\left(\frac{l_t}{k_t}\right)^{\phi}. \tag{3}$$

Assume the representative consumer, who acts in part as the goods producer, has utility denoted by u_t that is a log function of consumption goods denoted by c_t and leisure denoted by x_t, whereby with $\alpha \in R_{+}$, utility at time t is $u_t = \ln c_t + \alpha \ln x_t$. This is discounted over time by the subjective discount factor $\beta > 0$, with time preference ρ

set of parameters. The great bulk of the analysis found in textbooks of economic theory ... demand and supply analysis, the marginal analysis, and the Keynesian national income analysis all fall into this category."

defined as $\beta \equiv \frac{1}{1+\rho}$. Real output y_t is divided between the consumer's consumption and investment in new capital, denoted by i_t, such that

$$i_t = k_{t+1} - k_t (1 - \delta_k). \tag{4}$$

The time endowment of $T = 1$ each period is divided between labor and leisure:

$$T = 1 = l_t + x_t. \tag{5}$$

The consumer rents out labor, which equals $l_t = 1 - x_t$, and the accumulated capital k_t, with the resulting income net of capital investment spent on consumption goods as in the following budget constraint:

$$w_t (1 - x_t) + r_t k_t - c_t - k_{t+1} + k_t (1 - \delta_k) \geq 0. \tag{6}$$

Given k_0, the household's recursive problem is

$$v(k_t) = \underset{c_t, x_t, k_{t+1}}{Max} \{\ln c_t + \alpha \ln x_t + \beta v(k_{t+1})\}, \tag{7}$$

subject to equation (6), with the household equilibrium conditions given in Appendix A as are well-known for the Ramsey (1928) problem.

Note that restating the model completely in nominal terms may appeal to many since then with P_t the nominal price of goods, and with nominal wage defined as $W_t \equiv w_t P_t$, it results that the relative price for the aggregate supply and aggregate demand analysis can equally be stated throughout as having a relative price of $\frac{P_t}{W_t} = \frac{1}{w_t}$. This is done simply by factoring P_t through the budget constraint, with nominal income minus nominal consumption and investment exceeding or equally zero:

$$w_t P_t (1 - x_t) + r_t P_t k_t - P_t c_t - P_t k_{t+1} + P_t k_t (1 - \delta_k) \geq 0. \tag{8}$$

It is clear that $\frac{P_t}{W_t}$ and $\frac{1}{w_t}$ are identical by definition and so interchangeable. The nominal version when graphed matches the axes found in Ackley (1969).

4 Aggregate Demand and Supply for Output

The tradition in economics is to graph aggregate demand (AD) and supply (AS) against either a nominal price of goods, call it P_t, the rate of change in P_t, or against the real interest r_t. Microeconomics taken strictly is in terms of only relative prices, which is the focus here. The price P_t is allowed to enter when made relative to another such dollar price so that it is in units of a relative price, which equates to "real" terms. The real interest rate is typically viewed as such a real price. The other factor input price, the real wage w_t, can be defined as a relative price in terms of the dollar price of labor, call it W_t, divided by the dollar price of goods, P_t, where $w_t \equiv \frac{W_t}{P_t}$. The real price of labor w_t is a relative price of labor in units of the amount of goods received for renting one unit of time (an hour for example).

Similarly, $\frac{P_t}{W_t} = \frac{1}{w_t}$ is the amount of time given up per unit of output. This is the relative price of goods in terms of time. Ackley (1969) for example graphs AD and AS in terms of this $\frac{P_t}{W_t}$, although without deriving these from an optimization problem (p. 436, Figure 10-2). The two factor input prices, in terms of r_t and the inverse of w_t, or P_t/W_t, offer alternative ways to define the relative price of output, y_t, in terms of its aggregate supply and demand. This can give an $AS - AD$ analysis within either the (r_t, y_t) or the $\left(\frac{P_t}{W_t}, y_t\right)$ dimensions. Along the balanced growth path but with zero growth in the dynamic economy, these dimensions become independent of time. That is because it is a stationary state of a dynamic general equilibrium model. With continual increase in productivity as in the neoclassical growth model, the time subscripts would need to remain for the constantly growth variables. However without continuous technological change, the dynamic model yields a stationary capital stock, along with consumption, output, and investment. Initially keeping time subscripts, the alternative formulations of $AS - AD$ are examined along with the comparative static exercise of technological change that is at the heart of growth theory.

4.1 Baseline Model with P/W as Relative Price

Consider using the ratio of the nominal price of goods to the shadow nominal price of labor, or $\frac{P_t}{W_t}$, with output graphed in $\left(\frac{P_t}{W_t}, y_t\right)$ dimensions. The comparative static exercise of the dynamic model is conducted by demonstrating how aggregate demand

and supply change when productivity changes. This approach involves first deriving consumption demand for goods as a function of w_t and k_t and then adding to it the stationary investment of maintenance in capital given by $\delta_k k_t$. The sum of consumption and investment, each as functions of w_t and/or k_t, gives aggregate demand y_t as a function of w_t and k_t that can be graphed in $\left(\frac{P_t}{W_t}, y_t\right)$ space, given k_t. Equation (4) implies that consumer investment along the balanced growth path (BGP) stationary equilibrium with zero growth is simply capital maintenance:

$$i_t = k_{t+1} - k_t\left(1-\delta_k\right) = k - k\left(1-\delta_k\right) = \delta_k k. \tag{9}$$

From the budget constraint (6), in the BGP equilibrium,

$$c = wl + \left(r - \delta_k\right)k. \tag{10}$$

Now dropping times subscripts along the BGP equilibrium, from equations (49) and (50) in Appendix A, the BGP the marginal rate of substitution between goods and leisure can be solved for x as

$$x = \frac{\alpha c}{w}. \tag{11}$$

With the time constraint that $l = 1 - x$, the leisure of equation (11) can be substituted into equation (10) to get $c = w\left(1 - \frac{\alpha c}{w}\right) + \left(r - \delta_k\right)k$. From equations (49), (51) and (52) in Appendix A, the Ramsey (1928) equilibrium of $r - \delta_k = \rho$ results. Substitute this also into $c = w\left(1 - \frac{\alpha c}{w}\right) + \left(r - \delta_k\right)k$, solve for $c\left(w,k\right)$, and write the consumption demand as a function of w and k :[2]

$$c_t = \left(\frac{1}{1+\alpha}\right)\left(w + \rho k\right). \tag{12}$$

Adding consumption and investment of $\delta_k k$ together gives the aggregate demand for output $y\left(w,k\right)$ as denoted by AD as a function of w and k :

[2]This consumption function compares directly to Friedman (1957), in which $c = by_{perm}$, where b is a constant that here is derived as $1/\left(1+\alpha\right)$, and permanent income y_{perm} is here derived as the wage flow on the full time endowment and the rental flow on the capital wealth, $w + \rho k$. Friedman (1957) uses a two-period framework to conceive his consumption function while Ramsey 's(1928) is an infinite horizon.

$$AD : y = c + i = \left(\frac{1}{1+\alpha}\right)(w + \rho k) + \delta_k k = \frac{1}{1+\alpha}\left\{w + \left[\rho + \delta_k\left(1+\alpha\right)\right]k\right\}. \quad (13)$$

The aggregate supply of output as denoted by AS is derived by substituting the labor demand from the marginal product of labor into the production function for labor in equation (1). Solving for the BGP labor demand from the marginal product of $w = \phi A\left(\frac{l}{k}\right)^{\phi-1}$, gives that

$$l = \left(\frac{\phi A}{w}\right)^{\frac{1}{1-\phi}} k. \quad (14)$$

Substituting l from equation (14) into the BGP form of the production equation (1) and solving for output $y(w,k)$ as a function of w and k, the aggregate supply of goods is

$$AS : y = (A)^{\frac{1}{\phi}}\left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}} k. \quad (15)$$

Solving for $\frac{1}{w} \equiv \frac{P}{W}$ within each of the AD and AS equations, (13) and (15) gives equations for graphing $AS - AD$ in $\left(\frac{P}{W}, y\right)$ dimensions, given the capital stock k:

$$AD : \frac{P}{W} = \frac{1}{(1+\alpha)\,y - \left[\rho + (1+\alpha)\,\delta_k\right]k}; \quad (16)$$

$$AS : \frac{P}{W} = \frac{(y)^{\frac{1-\phi}{\phi}}}{\phi\,(A)^{\frac{1}{\phi}}\,k^{\frac{1-\phi}{\phi}}}. \quad (17)$$

The intersection of the AS and AD curves gives the solution for the equilibrium $P/W \equiv 1/w$, which in turn implies the equilibrium real wage w, given the capital stock k. Appendix B solves for the BGP k as a function of w by setting aggregate supply equal to aggregate demand. The real wage is then solved in closed-form along with the capital stock.

4.2 Baseline Example AS-AD with Consumption Demand

For the baseline example, let $\alpha = 1$, $\phi = 0.5$, $\rho = 0.05$, and $\delta_k = 0.03$. Figure 1 graphs the equilibrium $AS-AD$ curves of equations (16) and (17) (solid lines), given the equilibrium k. Here it results that $k = 1$, the relative price of output equals $1/w \equiv P/W = 6.67$ and the level of output is $y = 0.12$.

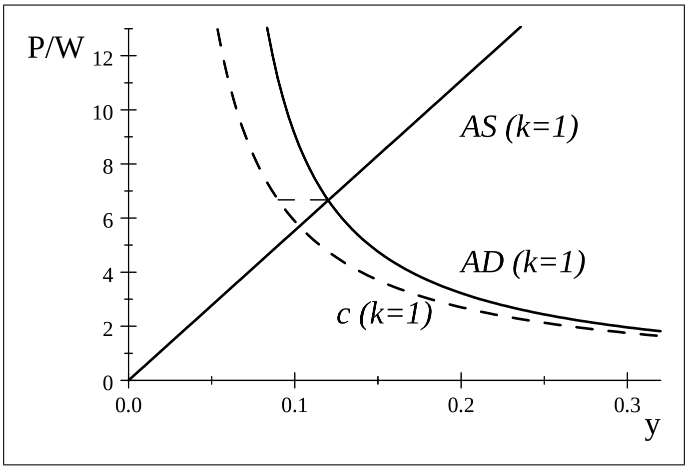

Figure 1: AD and AS, Consumption c, Investment.

To show the composition of aggregate demand as the sum of consumption demand and investment, Figure 1 also graphs in the dashed black curve the consumption demand of equation (12), as inverted to $\frac{P}{W} = \frac{1}{c(1+\alpha)-\rho k}$. The equilibrium consumption is found by the horizontal shift back in the AD curve to the consumption curve, by an amount equal to the equilibrium investment. The BGP capital maintenance of $\delta_k k$ is equal to $i = 0.03$, with $c = 0.09$ and $y = c + i = 0.12$.

As a calibration, this baseline example is reasonable in that c/y is 0.75, i/y is 0.25, and $k/y = 8.3$, all of which are found within the range for standard DSGE models. Setting the labor share of output at $\phi = 0.5$ splits the difference between the two main alternatives of $\phi = 0.67$ as is most common and $\phi = 0.33$ as is suggested by Mankiw et al. (1992). Here we are walking the line inbetween for two reasons. In extension to human capital the smaller share can be justified. Second, there is a problem with $\phi > 0.5$ in that this produces an upward-sloping but concave marginal cost of supply. This goes against notions of an increasing marginal cost as output increases, with the extreme version of this increasing marginal cost long used as a "vertical long run AS curve" in textbooks that results with $\phi = 0$. With $\phi < 0.5$, the upward-sloping supply curve is convex and consistent with an increasingly higher marginal cost as output rises, which is more plausible from a microfoundation perspective. Setting $\phi = 0.5$ avoids this debate, allows a closed-form solution for the Marshallian aggregate demand curve found in the subsequent Section 5.1, gives a linearly upward-sloping AS curve, and still allows the results of productivity increases and tax rate decreases to be qualitatively robust to alternative calibrations. For example using either of the above alternatives for ϕ gives the

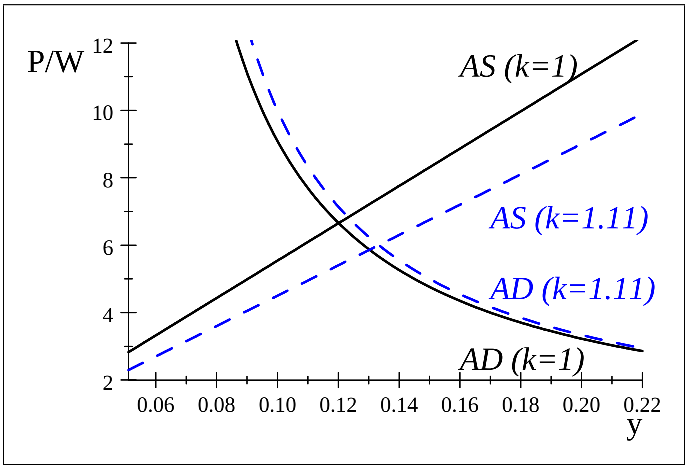

Figure 2: $AD - AS$, with $A = 0.19$ (solid), $A = 0.20$ (dashed blue).

same qualitative results for supply-side effects from increases in productivity or reductions in income tax rates.

5 Increase in Productivity

Given the same example, now let A rise from 0.19 to 0.20, a 5.26% increase. Figure 2 graphs the AS and AD in baseline (solid black) and the AS and AD with the productivity increase (blue dashed). The productivity increase causes capital k to increase from 1.0 to 1.11, or by 11%, which shifts out each the AS and AD curves according to equations 16 and 17. Output y rises to 0.133 from 0.12 and the real wage rises to 0.167 from 0.15, both 11% increases. As supply shifts out by more than demand, the relative price of output $1/w \equiv P/W$ falls to 6 from 6.67, an 11% decrease.

The productivity increase affects the marginal cost represented by the AS curve both through the increase in A itself in equation (17) and by the increase in the capital stock. The AD of equation (16) is affected only by the increase in the capital stock. The compound effects on the AS curve cause a significantly larger shift out in aggregate supply than in aggregate demand, giving the heart of the supply-side effect from a productivity increase: a net shift out in the AS.

The real wage rises as w rises from 0.15 to 0.167 by 5.26%, the same as the productivity factor, as P/W falls from 6.7 to 6. The labor stays constant at $l = 0.4$, so that y/l rises, and the capital stock and output rise by the same 11% so that y/k and r are constant. The decrease in P/W illustrates the Harberger (1998) proposition that the equilibrium

Variables	Productivity Change					
	Baseline Prod.	$\triangle 10\%$	$\triangle 10\%$	$\triangle 10\%$	$\triangle 10\%$	$\triangle 10\%$
y	0.1203	0.1456	0.1762	0.21.32	0.2579	0.3121
	-	21.00%	21.00%	21.00%	21.00%	21.00%
c	0.0902	0.1092	0.1321	0.1599	0.1935	0.2341
	-	21.00%	21.00%	21.00%	21.00%	21.00%
i	0.0301	0.0364	0.0440	0.0533	0.0645	0.0780
	-	21.00%	21.00%	21.00%	21.00%	21.00%
k	1.0028	1.2134	1.4682	1.7765	2.1495	2.6009
	-	21.00%	21.00%	21.00%	21.00%	21.00%
l	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000
	-	-	-	-	-	-
w	0.1504	0.1820	0.2202	0.2665	0.3224	0.3901
	-	21.00%	21.00%	21.00%	21.00%	21.00%
r	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600
	-	-	-	-	-	-
Tax Rev.	0.0160	0.0194	0.0235	0.0284	0.0344	0.0416
	-	21.00%	21.00%	21.00%	21.00%	21.00%

Table 1: Changes in Model Variables to Productivity Increases

marginal cost of output declines through a productivity increase.

More generally, the following Table 1 shows the changes in variables from a 10% step increase in productivity starting from the baseline calibration. This causes 21% increases in the variables that change, a slightly more than double the size of the productivity increase, just as in the example given above. Output, consumption, investment, capital wealth, and the real wage rate all growth equiproportionately. The employment rate l remains unchanged as income and substitution effects exactly offset each other and the real interest rate r is constant with zero growth in the intertemporal Euler equation (and for any positive exogenous rate of growth as well). As seen in the next section that incorporates taxes, another important effect given in the last row of Table 1 is that tax revenue also rises proportionately at the same rate. This row assumes equal labor and capital income tax rates of 20% as in the baseline example in Section 6.

Table 1 allows us to introduce the concept of the "productivity multiplier." Here we have a multiplier of a bit more than two, of the percentage change in variables due to a certain percentage change in productivity. This will be sensitive to the calibration. Qualitatively it shows that productivity raises output as in supply-side fashion in a quantitative way that can be computed and provide an area for further research. Productivity also raises tax revenue in the same way. This highlights that a government intervention that increases broad economic productivity without adding distortions would be able to pay for itself to the extent that the productivity increase raised tax revenue. This is true even without considering onwards effects on economic growth that could be computed by extending the model to make growth endogenous; see Gillman (2021b) for such a growth extension albeit without taxes.

Appendix C details how the productivity increase in the RDGE model affects the labor and capital markets, and production and utility functions, along with isoquants and isocosts. This analysis is seen to be standard although may suffer a lack of use given widespread uncertainty over the goods market. To further make ironclad the goods market analysis, consider it in terms of Hicksian versus Marshallian aggregate demand in the next Section 5.1.

5.1 In Hicksian vs. Marshallian AD

Figure 2 represents a Hicks (1946) aggregate demand curve along which utility is held constant at its equilibrium level, as is the capital stock k. It traces out slope of the equilibrium utility indifference curve in terms of the marginal rate of substitution between goods and leisure as given by equation (11), whereby $\frac{P}{W} = \frac{x}{\alpha c}$. Hypothetical changes in P/W along the AD curve correspond to induced changes in the choice of goods and leisure while utility remains constant.

Only at the intersection of the AD and AS curves, is the slope of the AD curve equal to the equilibrium P/W. At all other points along the AD curve, utility remains the same while the output level varies with P/W in accordance the maximal equilibrium utility level and the equilibrium k. That these other points are hypothetical under the condition that utility stays at its constant optimum level, the demand function is a Hicksian one with only substitution effects of relative price changes and with utility held constant. If the capital stock were allowed to change with P/W, then so would utility.

In this sense the AD curve of Figure 2 is a Hicksian utility-constant demand curve as described for example in Friedman (1976). A related concept of "real income" being held constant is inapplicable here since it is output (which equals income) that is changing along the demand curve. Rather, it is the capital stock, which is the agent's wealth, that is held constant at its equilibrium level such that utility is constant all along the AD curve. This AD curve is both a utility and capital-wealth constant curve that captures only prospective substitution effects with utility and capital constant as P/W changes.

The AS curve in turn graphs out the inverse slopes of the output production function in equation (1), in terms of P/W and y, as solved in the equilibrium of equation (15) and given the equilibrium k. Put differently, the AS curve exactly shows the magnitude of the inverse of the slope of the production function, or P/W, when this function is drawn in (y, l) space, with k held constant at it equilibrium level. Should the solution for k as a function of $W/P \equiv w$, or $k(w)$, be substituted in for k, then the AS curve becomes an amalgam of supply and demand in $\left(\frac{P}{W}, y\right)$ space, making that an infeasible representation of aggregate supply.

However, the AD curve alternately can be drawn by substituting in for $k(w)$ where k is solved as a function of w. Using this $k(w)$, the resulting AD curve goes through the same equilibrium point where the AS and AD curves intersect as does the Hicksian AD curve. But now wealth is changing as the relative price P/W changes, so that this curve includes both substitution and "income" effects in the form of changes in the capital stock wealth while the utility level also changes. As in Friedman (1976), this alternative Marshallian AD curve includes the full effect of the relative price change in P/W, or what he calls an "ordinary demand curve." With both substitution and income effects when P/W changes, both capital wealth and utility are changing along the demand curve.

To derive the Marshallian demand representation of the AD curve, the capital stock $k(w)$ is included in the form by which it depends on the real wage rate $w \equiv \frac{W}{P}$. As derived in Appendix B's equation (57), $k(w) = \frac{w}{(1+\alpha)\left[(A)^{\frac{1}{1-\phi}}\left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}} - \delta_k\right] - \rho}$. Substituting this $k(w)$ into equation (13), the Marshallian AD is given as a function only of $w = \frac{W}{P}$:

$$Marshallian\ AD : y = \frac{w\,(A)^{\frac{1}{1-\phi}}\left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}}}{(1+\alpha)\left[(A)^{\frac{1}{1-\phi}}\left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}} - \delta_k\right] - \rho}. \tag{18}$$

For the case of $\phi = 0.5$, the Marshallian AD of equation (18) can be solved for $P/W \equiv 1/w$ as given in the following:

$$Marshallian\ AD : \frac{P}{W} = \frac{y\left[(1+\alpha)\,\delta_k + \rho\right] + (A)^{\frac{1}{1-\phi}}(\phi)^{\frac{\phi}{1-\phi}}}{y\,(1+\alpha)\,(A)^{\frac{1}{1-\phi}}(\phi)^{\frac{\phi}{1-\phi}}}. \tag{19}$$

With the same baseline example, Figure 3 graphs both the baseline Hicksian AD

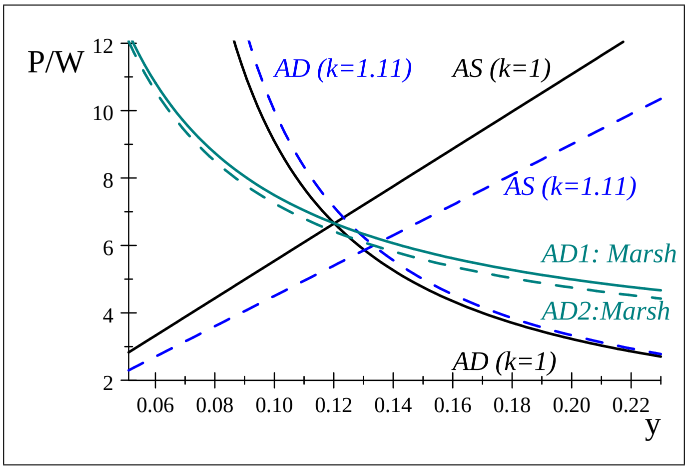

Figure 3: Marshallian $AD - AS$: Productivity Increase from $A = 0.19$ (Solid Green) to $A = 0.20$ (Dashed Green).

(black solid) and the Marshallian $AD1$ (green solid), which intersect the baseline AS curve (black solid) at the same point. The Marshallian $AD1$ (green solid) is flatter in $\left(\frac{P}{W}, y\right)$ space, and therefore more "price elastic", just as Friedman (1976) describes. With capital integrated in the AD as a function of W/P, the total function is more responsive to price change, making it more price elastic, and with each point along the AD showing a different level of utility.

Now consider the comparison between Hicksian and Marshallian demand when productivity increases. When A rises to 0.20, Figure 3 shows the Marshallian demand shifting back to $AD2$ (green dashed). It intersects the Hicksian AD and the AS curves (blue dashed) at the equilibrium of $y = 0.133$ and $P/W = 6$. The Hicksian AD shifting out as the result of a productivity increase is perhaps the more standard view of how aggregate demand reacts, although using either Marshallian or Hicksian versions result in the same equilibrium. In either case, there is a net shift out in the AS curve as productivity increases that represents a concept of supply-side economics.

5.2 In Alternative AS-AD with Real Interest Rate

The Keynesian tradition of focusing on the interest rate as the price for AS-AD and the New Keynesian tradition of using the intertemporal Euler condition for the AD curve can both be examined as well with a productivity increase. The AD with the real interest rate r as the price is still fully microfounded from the RDGE model, still shows a supply-side effect of the productivity increase, but also may be of less interest since the AD curve is horizontal, the AS curve is always linear regardless of calibration, and only the AS shifts when productivity increases.

Consider first the (r_t, y_t) dimensions that depend upon the real interest rate r_t. The first-order and envelope conditions of the consumer's optimization in equation (7), as given in Appendix A, imply the standard RDGE result in the BGP equilibrium that $r = \rho + \delta_k$. Alternatively with exogenous Solow (1956) growth where g is the stationary growth rate it results that $1 + g = \frac{1+r-\delta_k}{1+\rho}$ and so $r = \rho + \delta_k + g\,(1 + \rho)$. With μ the exogenous net rate of assumed technological change, a log approximation yields that the growth rate is fixed at $g = \frac{\mu}{\phi}$. Then with either $g > 0$ or $g = 0$, r is still fixed at a constant rate for a horizontal aggregate supply graph.

Here we keep the baseline economy of $g = 0$ and along the BGP conduct a comparative static increase in productivity. In the aggregate output dimensions of (r, y), this relation represents the horizontal, infinitely price elastic, aggregate demand denoted here by AD_r :

$$AD_r : r = \rho + \delta_k. \tag{20}$$

The aggregate supply curve, denoted by AS_r, is derived from the goods producer problem. Rearranging the marginal product of capital from the goods producer's equilibrium conditions, by which $r = (1 - \phi)\, A \left(\frac{l}{k}\right)^{\phi}$, the firm's labor demand l is given as a function of the capital stock:

$$l = \left(\frac{r}{(1-\phi)\,A}\right)^{\frac{1}{\phi}} k. \tag{21}$$

Now solve for l from the production function of equation (1), substitute that solution for l into equation (21), and solve for output y to derive the aggregate supply function, denoted by AS_r, as a function of r and k. Given the equilibrium k, then rearrange to solve this aggregate supply in terms of r for graphing in (r, y) dimensions.

$$AS_r \;:\; y = \frac{r}{(1-\phi)} k; \tag{22}$$

$$AS_r \;:\; r = \frac{y\,(1-\phi)}{k}. \tag{23}$$

For a given calibration, the equations (20) and (22) give the aggregate supply and demand along the BGP, AS_r and AD_r, which can be graphed with r as the vertical axis and y as the horizontal axis. For the baseline economy, with $\phi = 0.5$, $\alpha = 1$,

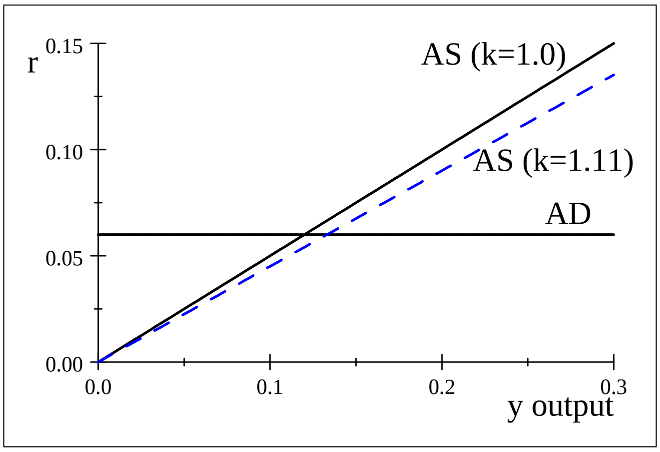

Figure 4: AD and AS of y graphed against r.

$\rho = \delta_k = 0.03$, and $A = 0.19$, equation (20) implies that $r = 0.03 + 0.03 = 0.06$, and equation (23) implies that

$$r = \frac{y\,(1-\phi)}{k} = (0.5)\,\frac{y}{k}. \tag{24}$$

Given that $k = 1$, Figure 4 graphs equation (24) as the AS line and $r = 0.06$ for the AD line, such that the equilibrium output is $y = 0.12$. Now let the productivity parameter A increase. This leaves the aggregate demand unchanged while aggregate supply shifts out (dashed line). Here A increases from 0.19 to 0.20 (5.26%); k rises from 1.0 to 1.11; output y from equation (22) is $y = \frac{0.06}{(0.5)}\,(1.11) = 0.133$. Both k and y rise by 11% such that the ratio y/k is unchanged at $\frac{0.1332}{1.11} = 0.12$, which is the same as when $A = 0.19$ and $\frac{y}{k} = \frac{0.12}{1} = 0.12$.

Figure 4 shows that the linear AS_r curve shifts out in this example, a supply-side shift albeit one with aggregate demand unchanged. This does not illustrate a fall in the relative price of output as Harberger (1998) emphasizes with a productivity increase. The marginal cost of output in the aggregate supply curve is always linear in this homothetic specification, without the possibility of a marginal cost rising at an increasing rate as in a convex marginal cost of production.

The change in A affects only the solution for the capital stock k, and not any other parameters that directly enter either the aggregate demand or aggregate supply functions in these dimensions. This means that the shift out in the AS_r line is only because of the capital stock increase. The only parameters that enter the AS_r function directly, ϕ, ρ and δ_k, are unaffected by a productivity increase.

6 Decrease in Tax Rates

Supply-side economics often is associated with productivity change that is at the heart of growth theory and real business cycle theory. Also the concept is invoked in considering how tax rate reductions increase incentives for economic activity that positively affect the economy. The idea that a net shift out in aggregate supply occurs with a productivity increase or a tax reduction has often been put forth. Here the introduction of a tax on income allows this to be quantified in the AS-AD aggregate goods market in this section.

6.1 Baseline Model Extension with Taxes

Now consider a flat tax on both labor and capital income that is the same rate, as in a flat tax income tax system. Denote the tax rates on labor and capital income by $\tau_l \in [0,1]$ and $\tau_c \in [0,1]$, respectively. The consumer pays the tax on income received by renting labor and capital to the firm, leaving the firm problem unchanged, with $w_t = \phi A \left(\frac{l_t}{k_t}\right)^{\phi-1}$ and $r_t = (1-\phi) A \left(\frac{l_t}{k_t}\right)^{\phi}$.

The consumer has the same capital accumulation and time constraints, with the only change being to the budget constraint. The income received from labor and capital is now $w_t l_t (1-\tau_l) + r_t k_t (1-\tau_c)$. The consumer also receives a lump sum transfer of tax revenue from the government that is denoted in real terms as Γ_t. This makes the consumer budget constraint:

$$w_t l_t (1-\tau_l) + r_t k_t (1-\tau_c) + \Gamma_t - c_t - k_{t+1} + k_t (1-\delta_k) \geq 0. \tag{25}$$

The tax income that goes to the government is transferred back to the consumer as the lump sum Γ with no government consumption of tax revenue. The government budget constraint is

$$w_t l_t \tau_l + r_t k_t \tau_c = \Gamma_t. \tag{26}$$

Given k_0, the consumer's recursive problem is

$$v(k_t) = \underset{c_t, x_t, k_{t+1}}{Max} \left\{ \ln c_t + \alpha \ln x_t + \beta v(k_{t+1}) \right\}, \tag{27}$$

subject to equation (25) and that $1 = l_t + x_t$. Substituting $l_t = 1 - x_t$ into the budget constraint and with λ_t the multiplier on the budget constraint, the first-order conditions and the envelope condition (with equality) are

$$c_t \quad : \quad \frac{1}{c_t} - \lambda_t = 0, \tag{28}$$

$$x_t \quad : \quad \frac{\alpha}{x_t} - \lambda_t w_t \left(1 - \tau_l\right) = 0, \tag{29}$$

$$k_{t+1} \quad : \quad \beta v' \left(k_{t+1}\right) - \lambda_t = 0, \tag{30}$$

$$k_t \quad : \quad v' \left(k_t\right) = \lambda_t \left[1 + r_t \left(1 - \tau_c\right) - \delta_k\right]. \tag{31}$$

On the BGP equilibrium,

$$r = \frac{\rho + \delta_k}{1 - \tau_c}. \tag{32}$$

The marginal rate of substitution between leisure and goods is equal to the tax-lowered shadow price of leisure:

$$\frac{\alpha c}{x} = w \left(1 - \tau_l\right). \tag{33}$$

From equations (25), (26), (32), and (33), the BGP consumption function denoted by $c\left(w, k, \tau_l, \tau_c\right)$ consequently is

$$c = \frac{1}{1 + \frac{\alpha}{(1-\tau_l)}} \left[w + \left(\frac{\rho + \delta_k}{1 - \tau_c} - \delta_k\right) k\right]. \tag{34}$$

Aggregate demand is given by adding capital maintenance to consumption:

$$AD : y = c + i = \frac{1}{1 + \frac{\alpha}{(1-\tau_l)}} \left[w + \left(\frac{\rho + \delta_k}{1 - \tau_c} - \delta_k\right) k\right] + \delta_k k \tag{35}$$

The effect of the taxes on aggregate demand is that the labor tax τ_l increases the effective time preference α, while the capital income tax τ_c raises the effective intertemporal market discount rate.

Aggregate supply is unchanged at

$$AS : y = (A)^{\frac{1}{1-\phi}} \left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}} k. \tag{36}$$

while the solution for the capital stock is altered by both taxes to become:

$$k = \frac{w}{\left(1 + \frac{\alpha}{1-\tau_l}\right) \left[(A)^{\frac{1}{1-\phi}} \left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}} - \delta_k\right] - \frac{\rho+\delta_k}{1-\tau_c} + \delta_k}. \tag{37}$$

Solving for $\frac{1}{w} \equiv \frac{P}{W}$ for the AD and AS equations, (13) and (15) gives equations for graphing $AS - AD$ in $\left(\frac{P}{W}, y\right)$ dimensions, given the capital stock k:

$$AD \quad : \quad \frac{P}{W} = \frac{1}{\left(1 + \frac{\alpha}{1-\tau_l}\right) y - k\left(\frac{\alpha\delta_k}{1-\tau_l} + \frac{\rho+\delta_k}{1-\tau_c}\right)} \tag{38}$$

$$AS \quad : \quad \frac{P}{W} = \frac{(y)^{\frac{1-\phi}{\phi}}}{\gamma (A)^{\frac{1}{1-\phi}} k^{\frac{1-\phi}{\phi}}}. \tag{39}$$

6.2 Baseline Example with Labor and Capital Income Taxes

With the baseline parameters of $\phi = 0.5$, $\rho = 0.03$, $A = 0.19$, $\alpha = 1$ and $\delta_k = 0.03$, assume in addition a 20% flat income tax of $\tau_l = \tau_c = 0.20$. Since $r = \frac{\rho+\delta_k}{1-\tau_c}$ and the marginal product of capital is $r = (1-\phi) A \left(\frac{l}{k}\right)^{\phi}$, then the solution for the labor to capital ratio is found by equating these latter two expression for r :

$$\frac{l}{k} = \left[\frac{\rho + \delta_k}{(1-\tau_c)(1-\phi) A}\right]^{\frac{1}{\phi}}. \tag{40}$$

Then using the marginal product of labor, $w = \phi A \left(\frac{l}{k}\right)^{\phi-1}$ and $\frac{l}{k}(\tau_c)$ in equation (40), the equilibrium real wage is lower because of the capital income tax τ_c :

$$w = \phi (A)^{\frac{1}{\phi}} \left(\frac{(1-\tau_c)(1-\phi)}{\rho + \delta_k}\right)^{\frac{1-\phi}{\phi}}. \tag{41}$$

In the example, $w = 0.5 (0.19)^{\frac{1}{0.5}} \left(\frac{(1-0.2)(1-0.5)}{0.03+0.03}\right)^{\frac{1-0.5}{0.5}} = 0.12$, as compared to 0.15 without the tax. The equilibrium capital stock decreases by almost half to

$$k(w) = \frac{0.12}{\left(1 + \frac{1}{(1-(0.2))}\right)\left((0.19)^{\frac{1}{1-0.5}}\left(\frac{0.5}{(0.12)}\right)^{\frac{0.5}{1-0.5}} - 0.03\right) - \frac{0.03+0.03}{1-(0.2)} + 0.03} = 0.53. \tag{42}$$

Aggregate supply and demand are given by

$$AD \quad : \quad \frac{P}{W} = \frac{1}{\left(1 + \frac{1}{(1-(0.2))}\right) y - (0.53)\left(\frac{1(0.03)}{1-0.2} + \frac{(0.03)+(0.03)}{1-0.2}\right)}; \tag{43}$$

$$AS \quad : \quad \frac{P}{W} = \frac{(y)^{\frac{1-0.5}{0.5}}}{0.5 (0.19)^{\frac{1}{1-0.5}} k^{\frac{1-0.5}{0.5}}}. \tag{44}$$

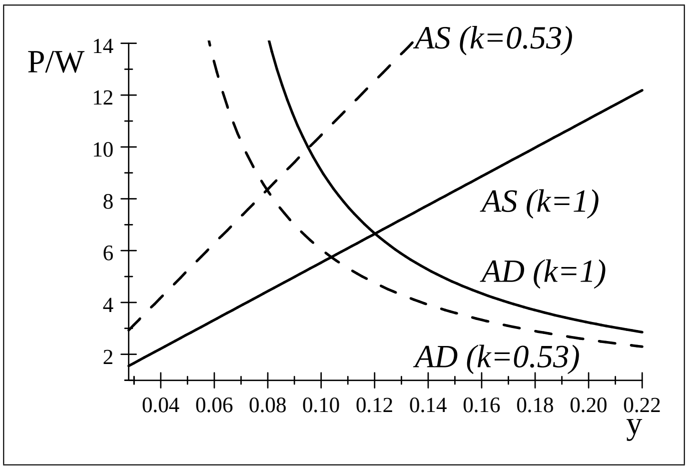

Figure 5: Supply-Side Economics: $AD - AS$ with Income Taxes; baseline $\tau_l = t_c = 0$ (solid) and with $\tau_l = t_c = 0.2$ (dashed).

Figure 5 graphs equations (43) and (44) to show that with $\tau_l = t_c = 0.2$, both the aggregate supply and demand shift back (dashed) compared to the baseline with zero taxes (solid). Here the AS shifts back by more than the AD, so that the relative price of goods P/W rises. Output falls by a 33% from $y = 0.12$ to $y = 0.08$; the relative price of goods rises by 24% from 6. 7 to 8.3; and the real wage falls by 20% from 0.15 to 0.12.

Conversely, Figure 5 implies that an equal flat tax rate reduction on both labor and capital income causes the aggregate supply of goods to shift out by more than the aggregate demand, as was the case for a productivity increase. Therefore both the flat equal income tax reduction and the productivity increase are found to be consistent with the supply-side concept of a net shift out in the aggregate supply.

6.3 Decrease in Labor Income Tax Rate to Zero

To see what drives the net shift out in aggregate supply, tax rate reductions can be isolated for each a labor income tax reduction and a capital income tax rate reduction. Consider first a labor income tax rate reduction to zero as compared to equal 20% tax rates on labor and capital income. Let $\tau_l = 0$ and $\tau_c = 0.2$ so that the tax remains only on capital income. Then the real wage remains the same at 0.12 and the capital stock increases from 0.53 to 0.61 :

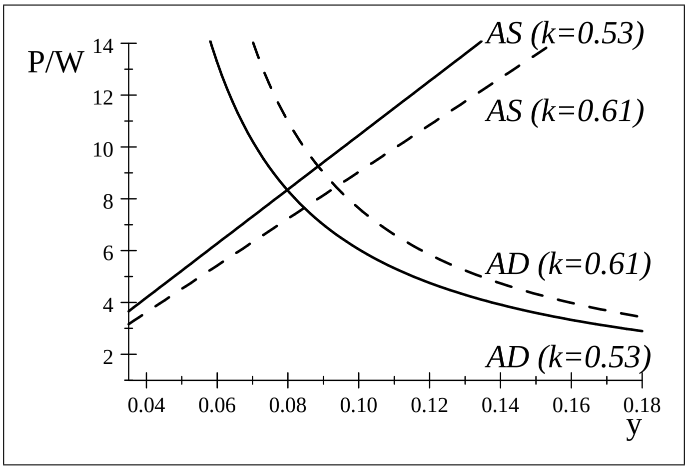

Figure 6: Supply-Side Economics: $AD - AS$ with Labor Income Tax Rate Reduction to Zero (dashed).

$$k(w) = \frac{0.12}{\left(1+\frac{1}{(1-(0))}\right)\left((0.19)^{\frac{1}{1-0.5}}\left(\frac{0.5}{(0.12)}\right)^{\frac{0.5}{1-0.5}} - 0.03\right) - \frac{0.03+0.03}{1-0.2} + 0.03} = 0.61. \quad (45)$$

Figure 6 shows that the AS and AD both shift out from the labor income tax reduction to zero. Since the real wage is unchanged so is the relative price P/W unchanged. The tax reduction raises output by 15% from 0.8 to 0.92. It is an example of supply-side economics although in this case of a labor income tax reduction both supply and demand shift out by the same amount horizontally. The strict supply-side economic concept of a net shift out in supply does not apply in this case. Both supply and demand are equal partners for the flat labor income tax reduction. This means that it must be a capital income tax rate reduction that causes a net shift out in supply.

6.4 Decrease in Capital Income Tax Rate to Zero

With a tax on only labor income and a reduction of the tax on capital income to zero, the supply-side economic concept of a net shift out in aggregate supply reemerges. Now let $\tau_l = 0.2$ and $\tau_c = 0$ so that there is a tax only on labor income. Using the same example, the real capital rental rate in equation (32) is now again $r = 0.06$ as with no taxes. As seen in equation (41), the real wage rises back up to its value in the baseline no-tax model, at $w = 0.15$. The only distortion is that τ_l increases the effective preference for leisure.

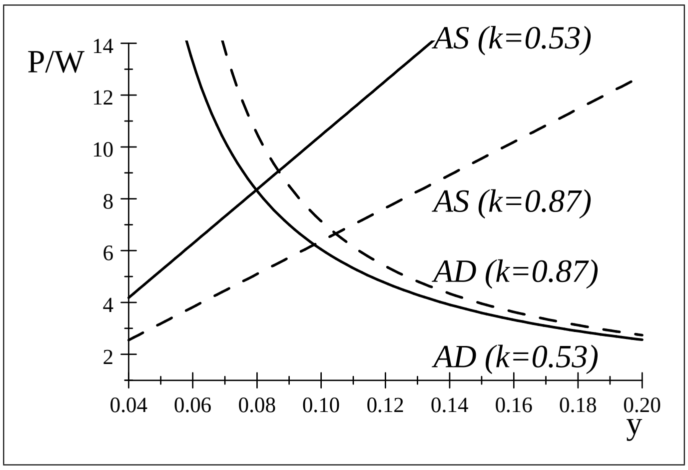

Figure 7: Supply-Side Economics: $AD - AS$ with Corporate Income Tax Rate Reduction to Zero (dashed).

The equilibrium capital stock then is

$$k = \frac{0.15}{\left(1 + \frac{1}{(1-(0.2))}\right)\left((0.19)^{\frac{1}{1-0.5}}\left(\frac{0.5}{(0.15)}\right)^{\frac{0.5}{1-0.5}} - 0.03\right) - \frac{0.03+0.03}{1} + 0.03} = 0.87. \qquad (46)$$

This capital stock increases by more that half to $k = 0.87$, as compared to 0.53 when $\tau_l = \tau_c = 0.2$.

Figure 7 graphs the baseline with $\tau_l = \tau_c = 0.2$ (solid) and the decrease of the capital income tax to zero, or $\tau_l = 0.2$ and $\tau_c = 0$ (dashed). Output rises 30% from 0.08 to 0.104 as the relative price of output falls from 8.3 to 6.7. There is a net shift out in aggregate supply. This is driven by the fact that the capital income tax rate reduction increases the real wage and the capital stock.

Figure 7 shows the net supply-side shift concept in $AS - AD$ terms of a capital income tax reduction to zero, which Lucas (1990) studies in more depth. It establishes within the RDGE model a net shift outwards in aggregate supply in this case and with a productivity increase. Together the tax reductions show that both labor and income tax reductions cause substantial increases in output through shifts out in both aggregate supply and aggregate demand.

6.5 Extension of Results

Consider reducing the labor and capital income tax rates from 0.5 down to 0 by incremental steps of 0.1. The following four tables show these effects for output and consumption in Table 2, for investment and capital wealth in Table 3, for employment and the wage rate in Table 4, and for the marginal product of capital and total tax revenue in Table 5. The Tables use the notation of τ_l/τ_c to show the effects of both the labor income tax rate τ_l and the corporate income tax rate τ_c in a side-by-side fashion.

Stated along the first column is the level of the labor income tax rate. Stated along the top row is the level of the corporate income tax rate. Moving down along any one column, each of these entries shows the percentage change in the particular variable from reducing the labor income tax by 0.1 while holding the corporate income tax rate at its given level. Moving across along any one row, each of these entries shows the percentage change in the particular variable from reducing the corporate income tax by 0.1 while holding the labor income tax rate at its given level.

In the first column of Table 2, the top-most entry shows that output is 0.033 when both tax rates equal 0.5. Keeping the τ_c at 0.5, and lowering τ_l to 0.4, to 0.3, to 0.2, to 0.1 and to 0, this first column shows that output rises by 15% with the first decrease of τ_l to 0.4 while $\tau_c = 0.5$. Output continues to rise but by decreasing percentages of 12%, 10%, 8%, and 7%, as τ_l continues to fall while holding $\tau_c = 0.5$.

Moving across the columns and comparing the decreases in τ_l for when $\tau_c = 0.4$, 0.3, 0.2, 0.1 and 0, it is clear that the percentage increases seen in the first column become marginally smaller as τ_c is lower. In sum, the magnitude of the increases in y remain nearly the same for decreases in τ_l, regardless of the level of τ_c. These findings for y are identical with those for consumption c that are found in the same Table 2.

Now consider reading the Table 2 to determine how decreases in the capital income tax rate τ_c affect output. Each row shows similar results to that of the effect of the labor tax in several ways: the percentage increase in output declines as the corporate tax rate level becomes lower; these increases are nearly the same regardless of the level of the labor income tax; and there is a slight decrease in the effect of τ_c going down as the level of τ_l goes down. This is just as with the labor income tax.

The key difference is that the percentage increase in output is larger from decreases in the capital income tax τ_c than in decreases in the labor income tax rate. This is true for each level of the labor income tax. These results also hold for consumption with one difference being that the increases in consumption are somewhat smaller than those in output from the corporate income tax decrease. As seen along the diagonal row in Table 2, the output change is about one-third to one-half bigger for the corporate income tax reduction than the labor income tax reduction at comparable levels of the income tax rates.

The different results for output and for consumption from the two tax rate decreases are due to different effects of the tax decreases on investment, since the sum of consumption and output equals investment. Table 3 shows explains the difference in the results by showing how both investment i ($i = \delta k$) and the capital stock k are affected by the decreases in τ_l and τ_c. The labor tax decrease, as seen by moving down the columns, causes exactly the same decrease in investment and capital as it does the decrease in output and consumption. All three variables increase at the same rate as the labor tax goes down, for any given level of the corporate income tax.

However the increases in investment and the capital wealth are more than three-fold larger for decreases in the corporate income tax rate than for the labor income tax when they are at similar levels. This is seen by comparing the percentage changes along the diagonal row in Table 3. A similar result is found along the diagonal in Table 2, but it is much more pronounced for the corporate tax rate reductions for investment and capital wealth.

The next Table 4 shows the tax reduction effects on labor employment l and the wage rate w. For employment, going down the columns it is seen that the labor income tax rate reductions produce increases in employment that decrease as the labor income tax rate is lower, with these ranging from 15% down to 7%. Comparing across the columns, the increases in employment are somewhat smaller as the capital income tax rate is lower.

For the capital income tax reductions, the change in employment is much smaller at about 2% for all levels of each of the taxes. It can be seen that the percentage employment increase goes up slightly as the corporate income tax rate falls, and that these increases are slightly smaller at lower levels of the labor income tax rate. Using the diagonal row, it shows that the labor income tax reductions cause a range from 7 down to 3 times the magnitude of the increase in employment from the capital income tax reductions.

As seen in equation (41), the wage rate in Table 4 is only affected by changes in the capital income tax rate. The table shows that the wage rate increases by a decreasing amount as the capital income tax is lowered, a percentage independent of the level of the labor income tax rate. The wage increases range from a 20% increase when τ_c drops to 0.4, to an 11% increase as it falls from 0.1 to 0.

Table 5 shows that similar to the wage rate changes, the marginal product of capital is affected only by the capital income tax reductions. This is seen also in equation (32). The marginal product decreases as τ_c falls, with a range from -17% to -10.

We also computed the effect of the tax rate reductions on total tax revenue, as defined by $\tau_l wl + \tau_c rk$. The second part of Table 5 shows that the tax revenue falls as tax rates go down, after a certain point that here is a 20% tax rate on both incomes. For higher tax rates, evidence of a Laffer curve is clear. Moving down along the diagonal row, the initial increase in tax revenues from the labor income tax reduction is less than from the capital income tax reduction. Once both tax rates have fallen to levels such that additional reductions in rates both cause lower tax revenue, it emerges that the labor income tax reduction causes a larger decrease in tax revenue than the capital income tax reduction.

The other important attribute about these tables is the relation between output and tax revenue, in Tables 2 and 5, respectively. As seen here, for equal tax rates of 20% in the baseline economy the output is 0.0802 and tax revenue is 0.0160. This implies that the share of tax revenue out of aggregate output is 19.95%. This is almost exactly equal to the share of annual US federal government spending in aggregate output for the 1960-2022 period, which was 19.96%.[3] This shows that the abstraction of only flat rate income taxes and the calibration yield precisely a realistic estimate of the share of stationary tax revenue out of output for the US economy, given the abstraction of only these sources of government finance.

6.6 Laffer Curves

The Laffer curves can be seen explicitly in the baseline model. Here we graph the change in tax revenues with one tax assumed to be zero and the other tax ranging from zero to 100%. This gives us a Laffer curve for both the labor income tax and the capital income

[3] U.S. Office of Management and Budget and Federal Reserve Bank of St. Louis, Federal Net Outlays as Percent of Gross Domestic Product [FYONGDA188S], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/FYONGDA188S, August 14, 2023.

Table 2: Changes in Output and Consumption to Tax Rate Changes

	Output					
τ_l/τ_c	0.5	0.4	0.3	0.2	0.1	0
0.5	0.0334	0.0410	0.0490	0.0573	0.0660	0.0752
	-/-	-/22.73%	-/19.38%	-/17.01%	-/15.24%	-/13.89%
0.4	0.0384	0.0471	0.0562	0.0656	0.0756	0.0860
	14.89%/-	14.78% / 22.61%	14.67%/19.26%	14.55%/16.88%	14.42%/15.12%	14.29/13.76%
0.3	0.0430	0.0526	0.0627	0.0732	0.0842	0.0957
	11.90%/-	11.81%/22.50%	11.70%/19.15%	11.59%/16.77%	11.48%/15.00%	/13.64%
0.2	0.0472	0.0578	0.0688	0.0802	0.0922	0.1046
	9.80%/-	9.71%/22.40%	9.62%/19.05%	9.52%/16.67%	9.42%/14.89%	9.32%/13.53%
0.1	0.0511	0.0625	0.0743	0.0866	0.0995	0.1128
	8.25%/-	8.17%/22.31%	8.09%/18.95%	8.00%/16.57%	7.91%/14.80%	7.81%/13.43%
0	0.0547	0.0669	0.0795	0.0926	0.1062	0.1203
	7.07%/-	7.040%/22.22%	6.92%/18.87%	6.84%/16.48%	6.75%/14.71%	6.67%/13.33%
	Consumption					
τ_l/τ_c	0.5	0.4	0.3	0.2	0.1	0
0.5	0.0292	0.0349	0.0404	0.0458	0.0512	0.0564
	-/-	-/19.22%	-/15.87%	-/13.46%	-/11.64%	-/10.22%
0.4	0.0336	0.0400	0.0463	0.0525	0.0586	0.0645
	14.89%/-	14.78%/19.11%	14.67%/15.75%	14.55%/13.34%	14.42%/11.52%	14.29%/10.09%
0.3	0.0376	0.0447	0.0517	0.0586	0.0653	0.0718
	11.90%/-	11.81%/19.00%	11.70%/15.64%	11.59%/13.23%	11.48%/11.41%	11.36%/9.97%
0.2	0.0413	0.0491	0.0567	0.0642	0.0714	0.0785
	9.80%/-	9.71%/18.90%	9.62%/15.55%	9.52%/13.13%	9.42%/11.30%	9.32%/9.86%
0.1	0.0447	0.0531	0.0613	0.0771	0.0771	0.0846
	8.25%/-	8.17%/18.81%	8.09%/15.46%	8.00%/13.04%	7.91%/11.21%	7.81%/9.77%
0	0.0479	0.0568	0.0656	0.0741	0.0823	0.0902
	7.07%/-	7.00%/18.73	6.92%/15.37%	6.84%/12.95%	6.75%/11.12%	6.67%/9.68%

tax.

The closed-form solution for tax revenue in general is $\tau_l wl + \tau_c rk$, where r in given by equation (32), k by equation (37), l by equation (40) and w by equation (41). The first term is graphed in Figure 8 and the second term in Figure 9.

Figure 8 shows the Laffer curve for the labor income tax rate assuming a zero capital tax rate and Figure 9 shows the corresponding Laffer curve for the capital income tax rate. These graph an extended set of data points that include the last column and the last row of Table 5, respectively. The difference in the figures from the Table 5 is that the tax rates go up to 100% instead of only to 50% in the table.

Two observations are first that the labor income tax Laffer curve has a peak at a higher rate than for the capital income tax rate. Second, the level of the tax revenue is higher for the labor income tax than for the capital income tax. Both peaks in the graphs are in the $50-60\%$ range. Given that the tax revenue share of GDP is in accordance with US data on the revenue share of GDP, along with various qualifications, this gives a baseline from which to view the point at which tax rate reductions reduce revenue but increase incentives for economic activity.

This is the first representation of a Laffer curve using the RDGE model with a closed-form solution that is linked precisely to the AS-AD analysis and is consistent with the

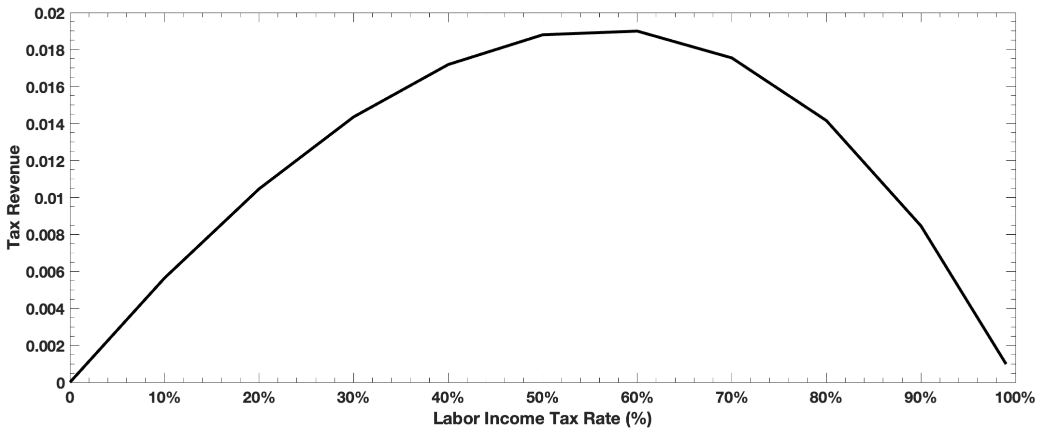

Figure 8: Laffer curve from changes in the labor income tax rate with the capital income tax rate equal to zero.

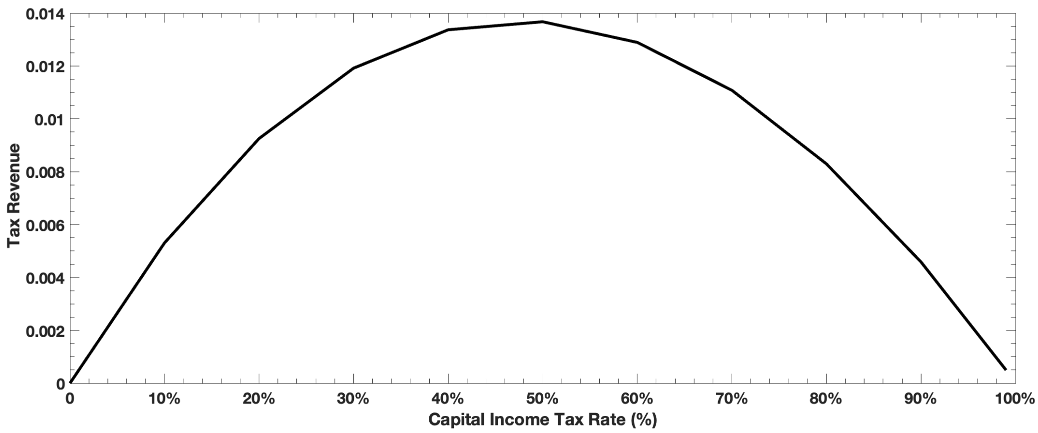

Figure 9: Laffer curve from changes in the capital income tax rate with the labor income tax rate equal to zero.

Table 3: Changes in Investment and the Capital Stock to Tax Rate Changes

	Investment					
τ_l/τ_c	0.5	0.4	0.3	0.2	0.1	0
0.5	0.0042	0.0062	0.0086	0.0115	0.0149	0.0188
	-/-	-/47.27%	-/39.28%	-/33.72%	-/29.65%	-/26.54%
0.4	0.0048	0.0071	0.0098	0.0131	0.0170	0.0215
	14.89%/-	14.78%/47.13%	14.67%/39.14%	14.55%/33.58%	14.42%/29.51%	14.29%/26.40%
0.3	0.0054	0.0079	0.0110	0.0146	0.0190	0.0239
	11.90%/-	11.81%/47.00%	11.70%/39.01%	11.59%/33.45%	11.48%/29.37%	11.36%/26.26%
0.2	0.0059	0.0087	0.0120	0.0160	0.0207	0.0262
	9.80%/-	9.71%/46.88%	9.62%/38.89%	9.52%/33.33%	9.42%/29.26%	9.32%/26.14%
0.1	0.0064	0.0094	0.0130	0.0173	0.0224	0.0282
	8.25%/-	8.17%/46.77%	8.09%/38.78%	8.00%/33.22%	7.91%/29.15%	7.81%/26.03%
0	0.0068	0.0100	0.0139	0.0185	0.0239	0.0301
	7.07%/-	7.00%/46.67%	6.92%/38.68%	6.84%/33.12%	6.75%/29.04%	6.67%/25.93%
	Capital					
τ_l/τ_c	0.5	0.4	0.3	0.2	0.1	0
0.5	0.1393	0.2051	0.2857	0.3820	0.4953	0.6267
	-/-	-/47.27%	-/39.28%	-/33.72%	-/29.65%	-/26.54%
0.4	0.1600	0.2354	0.3276	0.4376	0.5667	0.7163
	14.89%/-	14.78%/47.13%	14.67%/39.14%	14.55%/33.58%	14.42%/29.51%	14.29%/26.40%
0.3	0.1791	0.2632	0.3659	0.4883	0.6317	0.7977
	11.90%/-	11.81%/47.00%	11.70%/39.01%	11.59%/33.45%	11.48%/29.37%	11.36%/26.26%
0.2	0.1966	0.2888	0.4011	0.5348	0.6913	0.8720
	9.80%/-	9.71%/46.88%	9.62%/38.89%	9.52%/33.33%	9.42%/29.26%	9.32%/26.14%
0.1	0.2129	0.3124	0.4336	0.5776	0.7459	0.9401
	8.25%/-	8.17%/46.77%	8.09%/38.78%	8.00%/33.22%	7.91%/29.15%	7.81%/26.03%
0	0.2279	0.3343	0.4635	0.6171	0.7963	1.0028
	7.07%/-	7.00%/46.67%	6.92%/38.68%	6.84%/33.12%	6.75%/29.04%	6.67%/25.93%

traditional supply-side economics literature. Using the baseline calibration, it shows how decreasing the income tax rates yields higher revenue when these rates are very high (above $50 - 60\%$). And it shows that tax revenues decline as tax rates are decreased below the rate at which the Laffer curve peaks. This latter part of the curve would seem to apply to most developed economies, whose average tax rates are within these levels, although such a comparison is difficult to make given progressive marginal rate tax structures in those economies rather than the flat rate assumed here.

7 Discussion

Supply-side economics took off in earnest with the Laffer curve. The discussion has focused on considerations of productivity and the effect on incentives of tax distortions. In his 1977 Presidential address to the American Economic Association meetings, Klein (1978) emphasized as well the need for dynamic capital accumulation to be part of the model used to evaluate such effects: "The accumulation of capital contributes to the supply of goods and services. Indeed, investment demand now for new capital facilitates the implementation of the production process with the supply of factors of ever-increasing powers of productivity, thus making it possible to supply increasing amounts of goods

Table 4: Changes in Labor Time and the Wage Rate to Tax Rate Changes

	Labor Time					
τ_l/τ_c	0.5	0.4	0.3	0.2	0.1	0
0.5	0.2222	0.2273	0.2326	0.2381	0.2439	0.2500
	-/-	-/2.27%	-/2.33%	-/2.38%	-/2.44%	-/2.50%
0.4	0.2553	0.2609	0.2667	0.2727	0.2791	0.2857
	14.89%/-	14.78%/2.17%	14.67%/2.22%	14.55%/2.27%	14.42%/2.33%	14.29%/2.38%
0.3	0.2857	0.2917	0.2979	0.3043	0.3111	0.3182
	11.90%/-	11.81%/2.08%	11.70%/2.13%	11.59%/2.17%	11.48%/2.22%	11.36%/2.27%
0.2	0.3137	0.3200	0.3265	0.3333	0.3404	0.3478
	9.80%/-	9.71%/2.00%	9.62%/2.04%	9.52%/2.08%	9.42%/2.13%	9.32%/2.17%
0.1	0.3396	0.3462	0.3529	0.3600	0.3673	0.3750
	8.25%/-	8.17%/1.92%	8.09%/1.96%	8.00%/2.00%	7.91%/2.04%	7.81%/2.08%
0	0.3636	0.3704	0.3774	0.3846	0.3922	0.4000
	7.07%/-	7.00%/1.85%	6.92%/1.89%	6.84%/1.92%	6.75%/1.96%	6.67%/2.00%
	Wage Rate					
τ_l/τ_c	0.5	0.4	0.3	0.2	0.1	0
0.5	0.0752	0.0902	0.1053	0.1203	0.1354	0.1504
	-/-	-/20.00%	-/16.67%	-/14.29%	-/12.50%	-/11.11%
0.4	0.0752	0.0902	0.1053	0.1203	0.1354	0.1504
	-/-	-/20.00%	-/16.67%	-/14.29%	-/12.50%	-/11.11%
0.3	0.0752	0.0902	0.1053	0.1203	0.1354	0.1504
	-/-	-/20.00%	-/16.67%	-/14.29%	-/12.50%	-/11.11%
0.2	0.0752	0.0902	0.1053	0.1203	0.1354	0.1504
	-/-	-/20.00%	-/16.67%	-/14.29%	-/12.50%	-/11.11%
0.1	0.0752	0.0902	0.1053	0.1203	0.1354	0.1504
	-/-	-/20.00%	-/16.67%	-/14.29%	-/12.50%	-/11.11%
0	0.0752	0.0902	0.1053	0.1203	0.1354	0.1504
	-/-	-/20.00%	-/16.67%	-/14.29%	-/12.50%	-/11.11%

and services with inputs that are increasing at a somewhat slower rate. By focusing attention excessively on the "short run," in which the capital stock is timelessly held fixed by assumption only and not in reality, we have ignored the supply-side characteristics of investment demand (p.1)." Klein (1978) concludes with the need to incorporate this into the IS-LM framework, something that has never come to pass. The New-Keynesian models often likewise leave out the RDGE capital accumulation that is otherwise prevalent.

Considerations of related supply-side emphasis on revenue-neutral tax rate reform as in Laffer (1981) is complex because of the dynamics of capital accumulation. Lucas (1990) focused on this and led the way by showing how to incorporate the transition dynamics resulting from Ramsey (1928) capital accumulation into the welfare calculations of capital income tax reform. This laid the basis for applying it to tax revenue considerations. Azacis and Gillman (2010) applied the Lucas (1990) approach with a fully detailed tax structure to analyze tax reform in the Baltics. The latter showed how to achieve revenue neutrality, but this involves the full transitional dynamics that the current paper does not delve into or report. Revenue neutrality also requires consideration of endogenous growth that goes beyond the RDGE model. Such extensions

Table 5: Changes in the Marginal Product of Capital and Tax Revenue to Tax Rate Changes.

	Marginal Product of Capital					
τ_l/τ_c	0.5	0.4	0.3	0.2	0.1	0
0.5	0.1200	0.1000	0.0857	0.0750	0.0667	0.0600
	-/-	-/-16.67%	-/-14.29%	-/-12.50%	-/-11.11%	-/-10.00%
0.4	0.1200	0.1000	0.0857	0.0750	0.0667	0.0600
	-/-	-/-16.67%	-/-14.29%	-/-12.50%	-/-11.11%	-/-10.00%
0.3	0.1200	0.1000	0.0857	0.0750	0.0667	0.0600
	-/-	-/-16.67%	-/-14.29%	-/-12.50%	-/-11.11%	-/-10.00%
0.2	0.1200	0.1000	0.0857	0.0750	0.0667	0.0600
	-/-	-/-16.67%	-/-14.29%	-/-12.50%	-/-11.11%	-/-10.00%
0.1	0.1200	0.1000	0.0857	0.0750	0.0667	0.0600
	-/-	-/-16.67%	-/-14.29%	-/-12.50%	-/-11.11%	-/-10.00%
0	0.1200	0.1000	0.0857	0.0750	0.0667	0.0600
	-/-	-/-16.67%	-/-14.29%	-/-12.50%	-/-11.11%	-/-10.00%
	Total Tax Revenue					
τ_l/τ_c	0.5	0.4	0.3	0.2	0.1	0
0.5	0.00167	0.0185	0.0196	0.0201	0.0198	0.0188
	-/-	-/10.54%	-/6.12%	-/2.38%	-/-1.22%	-/-5.09%
0.4	0.0173	0.0188	0.0197	0.0197	0.0189	0.0172
	3.40%/-	2.03%/8.99%	0.33%/4.35%	-1.82%/0.19%	-4.65%/-4.07%	-8.57%/-8.99%
0.3	0.0172	0.0184	0.0188	0.0183	0.0168	0.0144
	-0.53%/-	-2.17%/7.19%	-4.26%/2.13%	-7.00%/-2.69%	-10.81%/-8.00%	-16.48%/-14.77%
0.2	0.0165	0.0173	0.0172	0.0160	0.0138	0.0105
	-3.92%/-	-5.96%/4.91%	-8.65%/-0.79%	-12.38%/-6.67%	-17.93%/-13.83%	-27.12%-24.32%
0.1	0.0153	0.0156	0.0149	0.0130	0.0099	0.0056
	-7.21%/-	-9.86%/1.92%	-13.53%/-4.84%	-19.00%/-12.57%	-28.06%/-23.47%	-46.09%/-43.29%
0	0.0137	0.0134	0.0119	0.0093	0.0053	0
	-10.77%/-	-14.40%/-2.22%	-19.81%/-10.85%	-28.77%/-22.34%	-46.62%/-42.65%	-100%/-100%

allow the effects on the growth rate to be considered in the calculations of how to achieve revenue neutrality through alternative tax reforms. Azacis and Gillman (2010) include such endogenous growth through the Lucas (1988) human capital approach in analyzing alternative tax reforms and revenue neutrality, but without any linkage to aggregate supply and demand.

This paper links rigorous RDGE model computations of supply-side effects to AS-AD but is limited by using only the RDGE framework. Extending this to endogenous growth and consideration of transition dynamics would be an avenue to extend the analysis while also linking it to AS-AD analysis as would be appropriate in that extended framework. It can also qualify the supply-side effects of tax reform by having both human and physical capital such that labor income and capital income tax rate reductions could both increase wealth accumulation.

For example, this paper's RDGE results on the labor tax that show an equal shift out of both aggregate supply and demand are specific to the RDGE model in which growth is exogenous. The real wage stays constant in this exercise that forces the relative goods price of $P/W = 1/w$ to be constant after the decrease in labor income tax rates. This requires that AS and AD shift out by the same amount. Extension with Lucas (1988) human capital provides a second intertemporal capital Euler condition that makes the growth rate endogenous and potentially allows labor income tax reductions to cause a greater shift out in AS than in AD. The AS-AD analysis of this paper requires modification for the Lucas (1988) model, as seen in Gillman (2021b). To conduct AS-AD analysis in a stationary state in a Lucas (1988) extension, output can be normalized by the human capital stock, since y/h would be stationary with h the level of the human capital. The AS-AD in terms of output per unit of human capital would depend on the state variable of such an extended RDGE model, which would be the physical capital per unit of human capital, or k/h, instead of just the physical capital stock level. Further research could investigate the extended AS-AD analysis in terms of supply-side applications. This would qualify the efficaciousness of capital versus labor tax reform as compared to that presented here with the RDGE model.

In such extension, the RDGE optimality of a zero capital income tax no longer applies in general. For example, with human capital led endogenous growth added, Turnovsky (2000) and Azacis and Gillman (2010) show that the second-best optimum is equal flat rate taxes on labor income and on capital income. As in Lucas (2000), they assume that tax revenue is constrained to be a certain share of output. The endogenous growth result is in contrast to the seemingly ubiquitously accepted result that zero capital income tax rates are optimal, even though that only applies in the RDGE model without the assumption that government spending is a constant fraction of output as for example in Greulich et al. (2023). Thus, the extension to the optimality of tax reform can also be addressed in supply-side economics using AS-AD and this analysis is modified once Lucas (1988) endogenous growth is added.

The current study is further qualified by the implicit assumption that there is zero tax avoidance/evasion. Adding tax evasion would lead to a greater size of the tax base the lower is the tax rate. This expanded tax base would be on top of the natural expansion of the tax base due to greater economic activity. This has been modeled with the representative agent choosing to report a larger share of taxes the lower is the tax rate (Gillman and Kejak , 2014; Gillman, 2021a). A full supply-side analysis could be done as in this paper in such an extended framework although closed form solutions are not available if capital is included in the production of the tax evasion by a financial intermediary.

Methodologically, we note that a key to the derivation of AS-AD analysis involves the issue that Klein (1978) highlighted: the stationary equilibrium capital stock. We incorporate this in AS-AD conceptually by taking advantage of the Stokey and Lucas (1989) recursive framework to state the RDGE model. This formulation makes the value function to be maximized dependent upon the known state variable at the beginning of the current time period, which in the RDGE model is the current period capital stock. This means that in the stationary state at the current time period, the capital stock in that time period is known. Then the AS-AD can be stated with output as a function of the relative price of the time required per unit of output, given the equilibrium capital stock at the current time period. The formulation of AS and AD in the goods market, as well as the supply and demand in the labor market, is dependent upon the given current period capital stock. This makes the AS-AD analysis internally consistent with the recursive structure of the RDGE problem in state-space terms since the capital stock in the current time period is known by the agent at the beginning of period in the dynamic optimization problem.

Further, in the stationary equilibrium the capital stock is the same for all future time periods as are all other variables in the RDGE model. This means that all time subscripts can be dropped. The equilibrium capital stock is stationary and the comparative static analysis can be done with the fully dynamic RDGE model. When productivity rises or the tax rate falls and the capital stock rises as in an "income effect" in the microeconomic sense, in the RDGE model this is instead a wealth effect, given the dynamic framework of capital accumulation as based on the stock of capital. Going from one comparative static equilibrium along the stationary state to another during such supply-side economic changes in parameters, the given capital stock changes and this is a part of the equilibrium shift of both AS and AD functions that depend on P/W and the capital stock.

The related result to emphasize is that along the stationary equilibrium AD curve, the one point on this AD curve that represents the equilibrium with the AS curve is also the tangency point between the production function and the equilibrium utility level indifference curve. Along the rest of the stationary AD curve, it maps out how substitution would occur as the relative price P/W changes, *given a constant capital stock* at the current stationary equilibrium and given the constant equilibrium utility equilibrium level at the current stationary equilibrium. This makes our baseline AD curve a Hicksian demand curve with utility held constant. Each point on the AD curve equals the inverse of the slope of the corresponding utility indifference curve as P/W changes while holding capital constant; this slope in turn equals the marginal rate of substitution between labor and goods. At the equilibrium of AD and AS, the slope of the AD equals the inverse of the equilibrium real wage W/P and the marginal product of labor.

During the above supply-side economic increase in productivity, the net shift out in the AS curve appears to be more naturally presented in the Hicksian version of the AD curve. The Hicksian AD curve shifts out due to the increase in the capital stock of wealth. The Marshallian AD can shift back with the comparative static change since it is more interest elastic than the Hicksian AD, as is illustrated above. Ackley (1969, pp. 249–251) suggests an AS-AD with a relative price of P/W and also "sketches...a possible derivation of the consumption function from an analysis of the way in which a 'rational' consumer maximizes utility over time". He derives a version of the "permanent income" consumption function using utility maximization over the infinite horizon but without the production technology and without solving for consumption. The derivation of AS-AD with P/W as the relative price of aggregate output is shown to be linked with the permanent income theory of consumption in equation (12) and Appendix D. The latter illustrates a special case in which a productivity increase shifts up the consumption function along a 45 degree line. It provides what might be called a Ramsey cross that can be compared to Samuelson's original cross. It is less compatible with the recent New-Keynesian "Keynesian cross" since that applies only to shock deviations from the stationary state, but more compatible with the cross in Guerrieri et al. (2022) that is for a model without capital.

Other limitations of the paper are that it could be made more general with alternative utility and production functions. It could include money and the inflation tax through a cash-in-advance economy and analyze how reductions in the inflation tax compare to reductions in income tax rates within the AS-AD framework. Other tax examples using the Keynesian sticky price and monopoly mark-ups are possible by using the Chari et al. (2007) approach that shows how these simplify to tax wedges. Given these limitations, we have made clear that the RDGE microeconomic-based $AS - AD$ analysis can be used to analyze supply-side economics, with the RDGE model the foundation of modern mainstream dynamic macroeconomic paradigms.

Policy implications are that supply-side economics indeed imply a greater shift out in aggregate supply than aggregate demand for productivity increases and capital income tax reductions. This also means that any government policy that increases productivity will induce a supply-side type shift and increase in main macroeconomic activity. This policy would have to be weighed against its cost. Efficacious enhancement of productivity can achieve a productivity multiplier that can compare to tax reform and government expenditure multipliers. Tax reform policy may itself increase productivity through spillover effects of greater economic activity. Understanding fundamentals of these supply-side issues and how they interact could improve government policy substantially. For example, government spending that efficaciously helps complete markets and supply social insurance policy as broadly framed might be expected to increase aggregate productivity. The value of such policy could be gaged in terms of both the cost of financing it through taxes that decrease economic activity and by the increased tax revenue and welfare if the program increases productivity.

8 Conclusion

The paper shows how to derive and construct this AS-AD framework in the Ramsey (1928) model and then apply it to main supply-side economic issues. It quantifies how a productivity increase affects main macroeconomic variables and tax revenue in the stationary state. And then it shows these effects from a decrease in the labor income tax rate and the capital income tax rate. Results show that productivity increases and capital income tax rate reductions shift out aggregate supply by more than demand.

Further detailed analysis quantifies the effects of the productivity increases and tax rate reductions across a range. As productivity increases, the macroeconomic variables that change rise slightly more than double the percentage increase in productivity. We term this the productivity multiplier. Its magnitude depends upon the specific calibration and could be computed across a range of alternative models.

Tax rate reductions show that the gains in output and other variables are larger the higher are the level of the initial tax rates. Decreasing the tax rates continues to increase variables, albeit at a decreasing percentage rate as the level of the tax rate is lower. Once on the tax-revenue decreasing side of the Laffer curve, tax revenue declines at an increasing rate as tax rates are decreased. The quantitative results are dependent upon the baseline calibration while the qualitative results are robust to non-extreme values of the fundamental technology and utility parameters that are calibrated in a standard fashion. The results are limited to those of the RDGE model, and a broad set of extensions are suggested. These include evaluating revenue-neutral tax rate reductions using an extension to endogenous growth, computing the effects of transition dynamics on welfare from productivity increases and tax rate reductions, modelling explicitly tax evasion that increases the size of the tax base as tax rates decrease, and adding the inflation tax through extension to a monetary economy that allows comparison of inflation rate reduction to income tax rate reductions and their consequent increased economic activity. How tax reform and other government policy affect productivity is an additional area of research that would further invigorate supply-side policies.

The paper contributes an internally consistent dynamic theory of value whereby aggregate supply and aggregate demand intersect to imply the equilibrium relative price of the aggregate good within the most widely used dynamic basis of macroeconomics. It applies this to study supply-side economics. This sharpens the view of how policy can affect output through increasing the effective productivity of the economy and decreasing income tax rates while increasing capital wealth. It derives such policy experiments in the price-theoretic terms of microeconomics through the microfoundations of AS-AD analysis, which hitherto have alluded both the policy realm and political economy more broadly.

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Appendix

A Household Equilibrium Conditions

The representative household maximizes recursive utility given k_0 :

$$v(k_t) = \underset{c_t, x_t, k_{t+1}}{Max} \{\ln c_t + \alpha \ln x_t + \beta v(k_{t+1})\}, \tag{47}$$

subject to the budget constraint with multiplier λ_t:

$$w_t(1 - x_t) + r_t k_t - c_t - k_{t+1} + k_t(1 - \delta_k) \geq 0. \tag{48}$$

The first-order conditions with equality are

$$c_t : \frac{1}{c_t} - \lambda_t = 0, \tag{49}$$

$$x_t : \frac{\alpha}{x_t} - \lambda_t w_t = 0, \tag{50}$$

$$k_{t+1} : \beta v' \left(k_{t+1}\right) - \lambda_t = 0; \tag{51}$$

and the envelope condition is

$$k_t : v' \left(k_t\right) = \lambda_t \left(1 + r_t - \delta_k\right). \tag{52}$$

B Closed-Form Solution for the Capital Stock

The real interest rate r is fixed by the intertemporal marginal rate of substitution of consumption goods over time, in the zero-growth economy that results when A is constant. In particular, this gives that $r = \rho + \delta_k$, the well-known result of Ramsey (1928). Given from the firm problem that the marginal product of capital is $r = (1 - \phi) A \left(\frac{l}{k}\right)^{\phi}$, set equal these latter two expression for r and derive the equilibrium labor to capital ratio as

$$\frac{l}{k} = \left(\frac{\rho + \delta_k}{(1 - \phi) A}\right)^{\frac{1}{\phi}}. \tag{53}$$

Given the marginal product of labor as $w = \phi A \left(\frac{l}{k}\right)^{\phi - 1}$, insert l/k from equation (53) into the latter marginal product to solve for the stationary wage rate as a function of underlying preference and technology parameters:

$$w = \phi (A)^{\frac{1}{\phi}} \left(\frac{1 - \phi}{\rho + \delta_k}\right)^{\frac{1-\phi}{\phi}}. \tag{54}$$

The equilibrium capital stock is also a closed-form solution of the economy's preference and technology parameters. Using market clearing that implies that the aggregate supply of y equals its aggregate demand, set equal the aggregate output from each of the AD and AS equations (13) and (15) to get a function depending upon two unknown variables w and k.

Denoting the aggregate demand for goods by y^d, and the aggregate supply by y^s, goods market clearing along the BGP implies that $y^d = y^s$. Therefore let the net excess quantity of goods demanded relative to the total quantity of goods supplied be set equal to zero. Subtracting output from equation (15) from equation (13), goods market clearing implies:

$$y^d - y^s = \frac{wT + k\left[\rho + (1+\alpha)\,\delta_k\right]}{1+\alpha} - A^{\frac{1}{1-\phi}}\left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}} k = 0; \tag{55}$$

This gives one equation in the unknowns of k and w :

$$\frac{1}{1+\alpha}\left\{wT + \left[\rho + \delta_k\,(1+\alpha)\right]k\right\} = A^{\frac{1}{1-\phi}}\left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}} k. \tag{56}$$

The solution for k as a function of w, indicated by $k\,(w)$ is a closed form solution as given by

$$k\,(w) = \frac{wT}{(1+\alpha)\left[(A)^{\frac{1}{1-\phi}}\left(\frac{\phi}{w}\right)^{\frac{\phi}{1-\phi}} - \delta_k\right] - \rho}. \tag{57}$$

Insert the solution for w from equation (54) into equation (57) to derive the solution for capital:

$$k = \frac{\phi\,(A)^{\frac{1}{\phi}}\left(\frac{1-\phi}{\rho+\delta_k}\right)^{\frac{1-\phi}{\phi}} T}{(1+\alpha)\left(\frac{\rho+\delta_k}{1-\phi}\right) - \left[\rho + \delta_k\,(1+\alpha)\right]}. \tag{58}$$

A convenient characteristic is that k/w simplifies to

$$\frac{k}{w} = \frac{(1-\phi)\,T}{\rho\,(\delta+\phi) + \phi\delta_k\,(1+\alpha)}$$

The closed form solution for y directly follows. Using the aggregate demand in equation (13), the solution to w in equation (54) and the solution to k from equation (58), then the equilibrium y is

$$y = \frac{1}{1+\alpha}\left\{\phi\,(A)^{\frac{1}{\phi}}\left(\frac{1-\phi}{\rho+\delta_k}\right)^{\frac{1-\phi}{\phi}}\left[1 + \frac{T\left[\rho+\delta_k\,(1+\alpha)\right]\rho\,(1-\phi)}{(\alpha+\phi)\,(\rho+\delta_k) - \delta_k\alpha\,(1-\phi)}\right]\right\}. \tag{59}$$

For the example, with $\phi = 0.5$, $\rho = 0.03$, $A = 0.19$, $\alpha = 1$ and $\delta_k = 0.03$, by equation (54), $w = \phi\,(A)^{\frac{1}{\phi}}\left(\frac{1-\phi}{\rho+\delta_k}\right)^{\frac{1-\phi}{\phi}} = (0.5)\,(0.19)^{\frac{1}{0.5}}\left(\frac{0.5}{0.06}\right) = 0.15$; by equations (58) and (59), $k = 1$ and $y = 0.12$. The capital-output ratio is $k/y = \frac{1.00}{0.12} = 8.34$; the savings rate $\frac{i}{y}$ is $\delta_k k/y = (0.03)\,8.34 = 0.25$. When A rises to 0.20, then k rises to 1.1, the real wage rises to $w = (0.5)\,(0.20)^{\frac{1}{0.5}}\left(\frac{0.5}{0.06}\right) = 0.167$, output rises to 0.133, $k/y = 8.25$ and the savings rate is the same at $\delta_k k/y = 0.25$.

C Productivity Increase: Input Markets, Output, Isoquants

This RDGE labor market, which has been shown throughout the literature in various ways, can also be shown in a similar way with either a Hicksian or a Marshallian labor demand curve. Graphed in the spatial dimensions of the real wage (how many goods are received for a unit of time) and the labor time, (w, l), the Hicksian labor supply in the example is convex with equilibrium utility level and capital stock held constant. The Marshallian labor supply that uses the solution of $k(w)$ yields a vertical supply curve that is invariant to changes in w. The labor demand, as with the goods supply of the AS curve, is the same in either case.

The firm's labor demand from equation (2) is given as $l = \left(\frac{\phi A}{w}\right)^{\frac{1}{1-\phi}} k$, which can be graphed as

$$w = \gamma A \left(\frac{k}{l}\right)^{1-\phi}. \tag{60}$$

For the supply of labor, from equation (12) the consumption function is $c = \left(\frac{1}{1+\alpha}\right)(wT + \rho k)$. Given equations (49), (51), and (52), by which $r = \rho + \delta_k$, inserting this for r in the budget constraint equation (8) implies in stationary equilibrium that $c = wl + \rho k$. Equating the latter two consumption expressions provides a solution for the supply of labor l as a function of w:

$$l = \left(\frac{1}{1+\alpha}\right)\left(T - \frac{\alpha \rho k}{w}\right), \tag{61}$$

which can be rewritten by solving for w as

$$w = \frac{\alpha \rho k}{T - l(1+\alpha)}. \tag{62}$$

Figure 10 graphs these supply and demand for labor functions using the baseline example calibration (solid). The demand curve is Hicksian with utility held constant given the value of the capital stock. When productivity A rise to 0.20, the marginal product of labor shifts out as the capital stock rises from $k = 1$ to $k = 1.1$. The capital increase causes a shift back in the supply of labor. Figure 10 shows that the real wage increases to $w = 0.167$ while $l = 0.4$ is constant. The shift out in labor demand is exactly offset by the shift back in labor supply, as is well known for this homothetic utility and

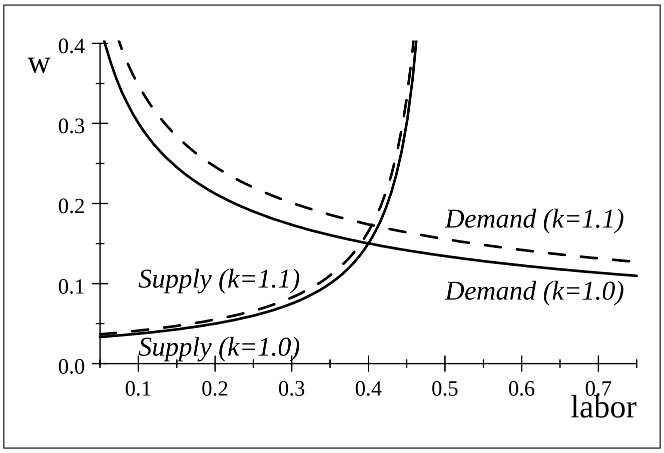

Figure 10: Labor Market

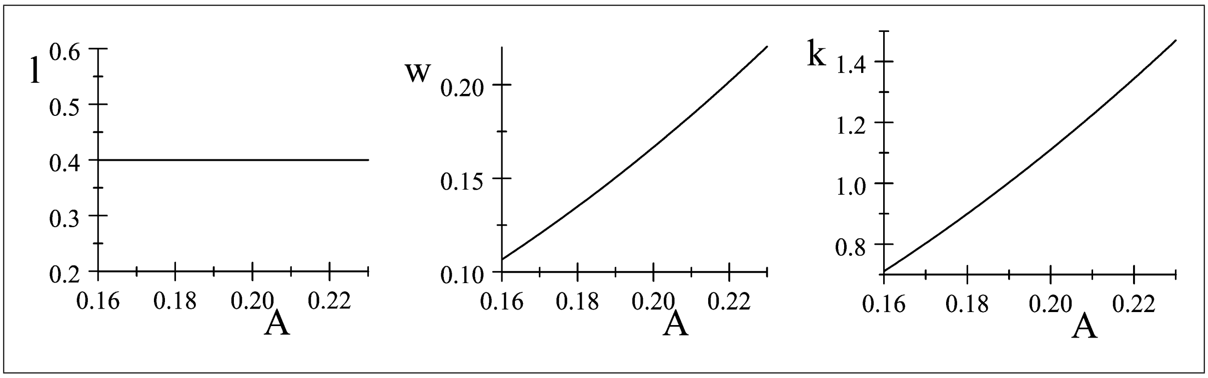

Figure 11: Labor, the Real Wage, and Capital Stock as Functions of A.

production function example.

Using the solution of $k(w)$ with the equilibrium capital stock and utility level changing as the real wage changes, the Marshallian labor supply that results from a change in A gives a vertical labor supply curve at $l = 0.4$. The income and substitution effects exactly offset each other.

Complementing Table 1, a supplemental graphical view of how productivity increases affect the macro variables of w and k along with l is given in Figure 11. This graphs the increase in $w(A)$ and $k(A)$ as productivity rises, with these functions derived from Appendix B equations (54) and (58) (center and right panels), respectively. In the left panel it shows that $l = l(A) = 0.4$ for all A, derived from equation (61) combined with Appendix B equation (57).

The capital demand is given by the firm's marginal product of capital in equation (3) as $r_t = (1-\phi)A\left(\frac{l_t}{k_t}\right)^{\gamma}$. The capital supply is fixed at $r = \rho + \delta_k$. Given the example calibration, $r = 0.06$. With $A = 0.19$, Figure 12 graphs the capital market with the

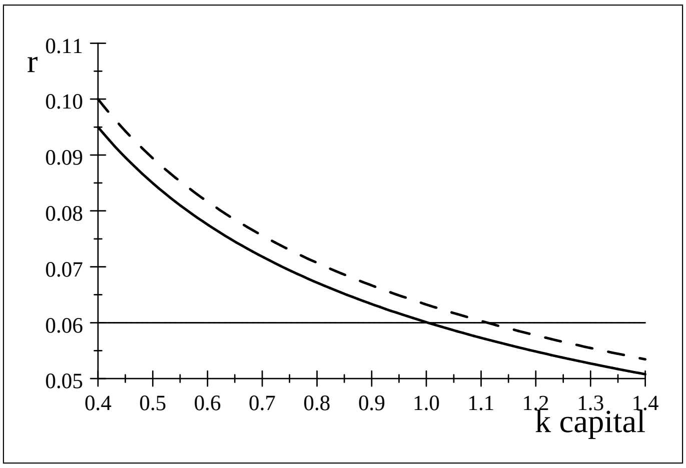

Figure 12: Capital Market

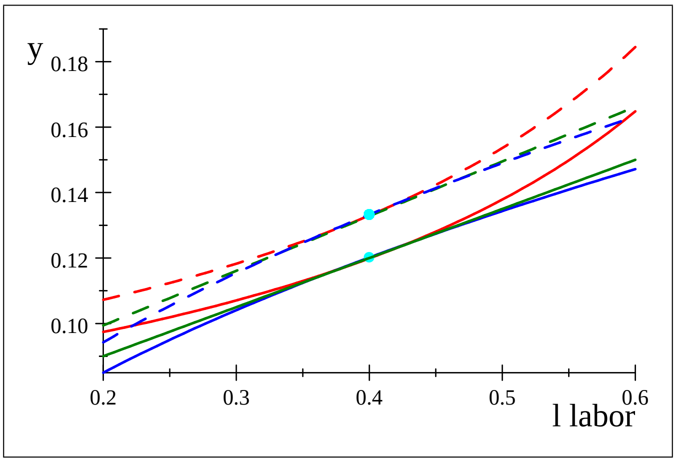

Figure 13: Production of y, Utility u; & $c = y - \delta_k k$.

demand shown (in the solid curve), given the equilibrium $l = 0.4$ and with supply shown in the horizontal line at $r = 0.6$. The equilibrium capital results at $k = 1$. When productivity rises so that A increases to 0.20, then l remains the same as seen in the labor market above. However the demand for capital shifts (dashed curve) because of the increase in A. This creates an equilibrium capital stock of $k = 1.1$.

The production function is $y_t = A\,(l_t)^{\phi}\,(k_t)^{1-\phi}$ and utility is $u_t = \ln c_t + \alpha \ln x_t$. Along the BGP with the example calibration, Figure 13 graphs the production function (solid blue) in (y, l) space taking as given the equilibrium $k = 1$:

$$y = 0.19\,(l)^{0.5}\,(1.0)^{0.5}\,. \tag{63}$$

For the optimal utility indifference curve from $u = \ln(c) + \ln(x)$, substitute in $c = y - i$, with $i = \delta_k k$, and $x = T - l$, and with $\alpha = T = 1$, and $\delta_k = 0.03$; then solve for the equilibrium u in terms of y and l. Figure 13 graphs the optimal indifference curve (solid red) as equation (64):

$$\begin{aligned} u^* &= \ln(y-0.03)+\ln(1-l); \\ -2.92 &= \ln(0.12-0.03)+\ln(1-0.4); \\ y &= \frac{e^{-2.92}}{(1-l)}+0.03. \end{aligned} \tag{64}$$

For the budget-profit line, $y = wl + rk$; this is both the consumer's budget constraint and the firm's profit line. Figure 13 graphs the example (solid green) from the following equation:

$$y = 0.15l + 0.06, \tag{65}$$

where $rk = 0.06$ is the vertical axis intercept of this line. These latter three equations are tangent at the equilibrium point of $l = 0.4$. An increase in productivity to $A = 0.20$ shifts up the production function, equilibrium utility and the slope of the budget line to $y = 0.20\,(l)^{0.5}\,(1.11)^{0.5}$, $y = \frac{e^{--2.784}}{(1-l)} + 0.03$ and $y = 0.167l + 0.06\,(1.1)$, respectively. This is seen in the dashed lines of Figure 13.

Figure 14 graphs the input market in (k, l) space for the isoquant, isocost, and input ratio. The equilibrium isoquant is $y = A\,(l)^{0.5}\,k^{0.5} = 0.12$, which is rearranged for graphing to $k = \left(\frac{0.12}{0.19}\right)^2 \frac{1}{l}$ (solid blue). The equilibrium isocost line is $0.12 = (0.15)\,l + 0.06k$, or $k = \frac{0.12}{0.06} - \frac{0.15}{0.06}l$ (solid green). The input ratio, from the firms equilibrium conditions (2) and (3) is that $\frac{w_t}{r_t} = \frac{\gamma A\left(\frac{l_t}{k_t}\right)^{\phi-1}}{(1-\phi)A\left(\frac{l_t}{k_t}\right)^{\phi}}$, so that $k = \frac{w}{r}l\frac{1-\phi}{\phi} = \frac{0.15}{0.06}l$ (solid black). When A increases to 0.20 (dashed lines), labor remains unchanged at $l - 0.4$ as capital rises from 1.0 to 1.1, and output rises to 0.133. The isoquant shifts out, the isocost line pivots upwards to the right and the input ratio pivots upward to the left.

D Consumption Theory Analogue to Ramsey Cross

Our equilibrium above for consumption implies that it is a fraction of permanent income. By equation (12), $c = \left(\frac{1}{1+\alpha}\right)(wT + \rho k)$, where $1/(1+\alpha)$ is the fraction and $wT + \rho k$ is the permanent income flow from the value of the endowed time and the equilibrium capital stock. However, it is also true that consumption can be equal to "full income (Becker, 1965)" of Tw.

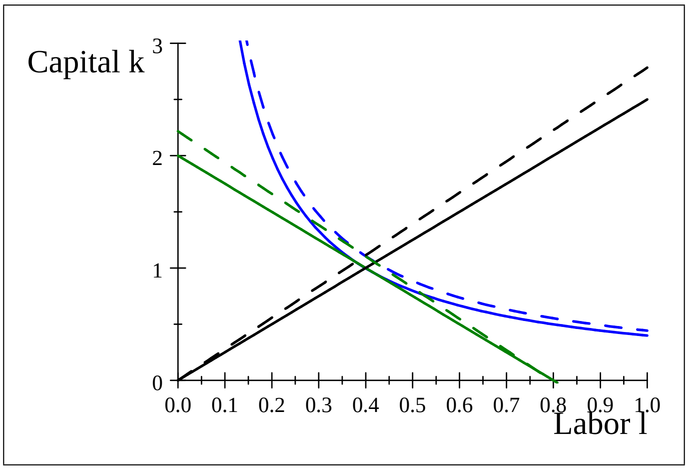

Figure 14: Isoquant, Isocost and Input Ratio with Baseline Model and a Productivity Increase.

This gives rise to a "Ramsey Cross" whereby a productivity increase causes a shift up in $c = \left(\frac{1}{1+\alpha}\right)(wT + \rho k)$, as it moves along the 45 degree line of $c = w$ in the case that $T = 1$. To see this first consider the following Proposition:

Proposition 1 *By equation (12), equilibrium consumption is a fraction of permanent income as given by* $c = \frac{1}{1+\alpha}(wT + \rho k)$*. If* $\alpha = \frac{\rho(1-\phi)}{\rho+\phi\delta_k}$*, then it is also true consumption* $c = Tw$*, and consumption is equal to "full income"* Tw*.*

Proof. If true, then permanent income equals w as follows:

$$
\begin{aligned}
c &= \left(\frac{1}{1+\alpha}\right)(wT + \rho k) = wT, \\
&\implies \alpha = \frac{\rho k}{Tw} \qquad (66)
\end{aligned}
$$

Substitute into equation (66) the equilibrium for w and k from equations (54) and (58) respectively to yield in reduction that $\alpha = \frac{\rho(1-\phi)}{\rho+\phi\delta_k}$. ■

With the same example calibration above for $\rho = 0.03$, $\phi = 0.5$, and $\delta_k = 0.03$, let $\alpha = 0.3333$ instead of 1.0 as assumed above. Indeed we could assume $\alpha = 0.3333$ above and all of the analysis and comparative statics for the goods, labor and capital markets would remain qualitatively the same. We use $\alpha = 1.0$ above for the baseline and less numerical complexity.

Corollary 2 *Consumption in the Ramsey model can also be written as* $c = a + bwT$ *with* $b < 1$*.*

Proof. With $c = \frac{1}{1+\alpha}(\rho k + wT)$, then $c = a + bwT$, where $a = \frac{\rho k}{1+\alpha}$ and $b = \frac{1}{1+\alpha}$. Given any positive leisure preference, $\alpha > 0$, then $b = \frac{1}{1+\alpha} < 1$. ■

This gives a RDGE version of the Keynesian consumption function. Assume as in the baseline calibration that all parameters are as above, $\rho = 0.03$, $\phi = 0.5$, $\delta_k = 0.03$ and $T = 1$, except α which is set at $\frac{1}{3}$. Then the capital stock of equation (58) gives an equilibrium of $k = 1.67$; it is higher than when $\alpha = 1$ since there is less equilibrium leisure, more labor, more output and more capital needed for that output. The real wage from equation (54) is independent of α and equal to 0.15. Output from equation (15) is now higher at 0.20. Labor from equation (14) is higher at $l = \frac{2}{3}$, so that leisure is lower at $\frac{1}{3}$. Now consider how the RDGE model implies a 45 degree line that is the other part of the cross diagram and that intersects with the $c = a + bwT$ line.

Corollary 3 *Given* $\rho = 0.03$, $\phi = 0.5$, $\delta_k = 0.03$, $T = 1$, *and* $\alpha = \frac{1}{3}$, *then* $k = 1$, $a = \frac{0.03}{1+\frac{1}{3}}$, $b = \frac{1}{1+\frac{1}{3}}$ *and* $c = \frac{0.03}{1+\frac{1}{3}} + \frac{1}{1+\frac{1}{3}}w$; *also* $c = w$. *These latter two consumption equations intersect at the equilibrium consumption of* 0.15; *a* 5.25% *increase in productivity of* A *from* 0.19 *to* 0.20 *shifts up the* $c = a + bw$ *equation along the* 45 *degree line of the* $c = w$ *equation to reach an* 11% *higher equilibrium* $c = 0.1667$.

Proof. With $\rho = 0.03$, $\phi = 0.5$, $\delta_k = 0.03$, $T = 1$, $\alpha = \frac{1}{3}$, the capital stock of equation (58) is $k = 1.6713$; $w = 0.15$ in equation (54) (independent of α). Since $c = 0.0376 + (0.75)(0.15) = 0.15 = w$, then $c = a + bwT = wT$ is established for $A = 0.19$. With $A = 0.2$, then $k = 1.852$, $w = 0.1667$, and $c = 0.042 + (0.75)(0.1667) = 0.1667 = w$. Therefore c rises as A increases, while c still equals w. The line $c = wT$ has a slope of one in (c, wT) dimensions, an angle of 45 degrees, and $c = a + bwT$ in the same (c, wT) dimensions has an slope of $b = 0.75 < 1$; as a increases from 0.376 to 0.042 the line $c = a + bwT$ in (c, wT) dimensions shift ups along the $c = wT$ line in (c, wT) dimensions; the $c = wT$ line does not shift. ■

Figure 15 illustrates Corollary 3 diagramatically. We could add equilibrium savings, denoted by s, to the wT income line to get consumption plus savings. Since savings equals investment, $s = i = \delta_k k$, then $c + s = wT + \delta_k k$. On the vertical axis we can add investment to consumption to get total output: $c + i = y$.

For the same calibration, Figure 16 illustrates the shift up in the cross intersection for $c + i$ when productivity increases (blue dashed). Here the aggregate output line has

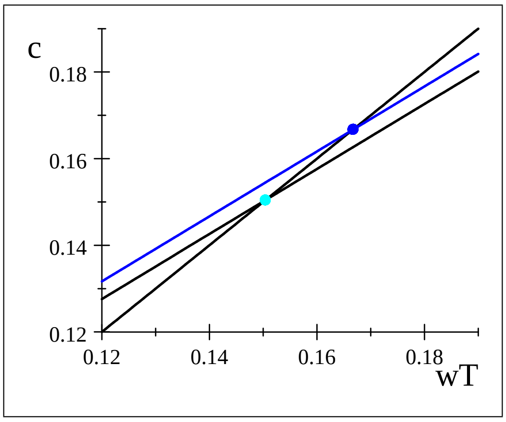

Figure 15: The Ramsey Cross Consumption Theory: Shift up in $c = a + bwT$ along $c = wT$ with an Increase in Productivity A.

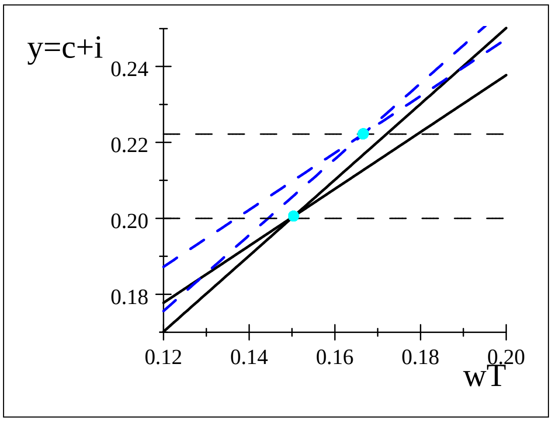

Figure 16: Aggregate Output and Income: $c+i = c+s$: Cross shifts up with an Increase in Productivity A.

a slope of $\frac{1}{1+\alpha} = 0.75$, with the equation of $c + i = \left[\frac{\rho k}{1+\alpha} + \left(\frac{1}{1+\alpha}\right) Tw\right] + \delta k$ as graphed against wT on the horizontal axis. The aggregate income line has a slope of one, with the equation of $c + s = wT + \delta_k k$ also graphed as a function of wT on the horizontal axis. When productivity rises from 0.19 to 0.20, both lines shift up to give a higher wT and aggregate output relative to the baseline (solid black). Aggregate output and income rise from 0.2 to 0.222 as wT rises from 0.15 to 0.167. This is the same wage increase as in the baseline above since w is independent of α in equation (54).

The "propensity to consume" out of "current income" wT is the constant parameter b in the $c = a + bwT$ form. Dependent only upon leisure preference α, this propensity is constant when an increase in A causes endogenous increases in the real wage w and the capital stock k. The shift upwards in the cross might be accomplished by the government causing an increased productivity of investment, such as through investment in human and physical capital investment infrastructure or through bank insurance policy reform after a deep banking crisis-induced recession. Here a 5% productivity increase leads to an 10% capital stock increase and a similar sized increase in output so that the example productivity "multiplier" here is about two. This gives room for efficient government social insurance that gives a net increase in productivity after including any increased tax distortions in a more general model.