Welfare Cost of Inflation in a Stochastic Balanced Growth Model

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Abstract

There is a large and growing literature on the welfare cost of inflation. However, work in this area tend to find moderate estimates of welfare gains. In this paper we reexamine welfare costs of inflation within a stochastic general equilibrium balanced growth model paying a particular attention to recursive utility, portfolio balance effects, and monetary volatility and monetary policy uncertainty. Our numerical analysis shows that a monetary policy that brings down inflation to the optimum level can have substantial welfare effects. Portfolio adjustment effects seem to be the dominant factor behind the welfare gains.

Keywords: Inflation; Monetary policy; stochastic growth models.

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Abstract

There is a large and growing literature on the welfare cost of inflation. However, work in this area tend to find moderate estimates of welfare gains. In this paper we reexamine welfare costs of inflation within a stochastic general equilibrium balanced growth model paying a particular attention to recursive utility, portfolio balance effects, and monetary volatility and monetary policy uncertainty. Our numerical analysis shows that a monetary policy that brings down inflation to the optimum level can have substantial welfare effects. Portfolio adjustment effects seem to be the dominant factor behind the welfare gains.

1   Introduction

Central to the policy question of whether governments should aim to reduce inflation is the magnitude of potential gains from doing so. As a result there is a large and growing literature measuring the welfare costs of inflation. However, work in this area tend to find moderate estimates of welfare gains [see Cooley and Hansen (1991), Braun (1994), Lucas (1994), Jones and Manuelli (1995), and Dotsey and Ireland (1996)]. It is found that reducing inflation from a baseline of 4% to socially optimum levels entails a welfare gain no more than 1% of real income. In a well known paper, Lucas (2000) puts the welfare costs of reducing inflation solidly in this range. Nevertheless, these estimates are not consistent with the general public's perception of inflation: Public opinion surveys still document a profound dislike of inflation. For example, Shiller (1997) reports that inflation is the main concern of over 80 percent of those surveyed.

The discrepancy between the literature and public opinion may reflect the limitations of the models. For example, the existing literature has paid a scant attention to welfare costs of inflation associated with uncertainty, particularly monetary volatility and monetary policy uncertainty. As Lucas (2000) pointed out, a large body of the literature "misses important costs of inflation that are thought to arise from price or inflation rate *variability*" (p.258, italics in original). In an uncertain world, monetary policy has level as well as variability effects. It is known that uncertainty increases precautionary savings, making agents more willing to substitute funds from this period to the next period [see Den Haan (1990)]. This then implies that agents will hold the satiation level of money balances at a lower rate of deflation. Moreover, money superneutrality does not carry into the uncertainty case as shown by Danthine, Donaldson, and Smith (1987). Works with uncertainty such as Den Haan (1990) and İmrohoroĝlu (1992) find welfare estimates of inflation that are substantially larger

than deterministic models suggest. However, the optimum inflation literature has largely ignored portfolio balance effects, monetary volatility and monetary policy uncertainty. Changes in the inflation rate alter nominal returns of assets and thereby induce optimizing individuals to adjust their optimal portfolio shares. Such adjustments can generate growth effects and hence welfare effects.[1] Monetary volatility is an additional source of uncertainty and hence magnifies aggregate variability in an economy, in particular, inflation variability. This may have nontrivial economic growth effects as demonstrated by Dotsey and Sarte (2000), portfolio adjustment and thereby welfare effects. As for monetary policy uncertainty, it relaxes the assumption that households are perfectly informed about the distribution of money growth [Stulz (1986)]. This implies that the mean of money growth is not observable and agents therefore try to learn about the monetary policy rule. This may be expected to add another friction as the predictive distribution over money growth has a higher variance than the variance of money growth.

Apart from accounting for portfolio balance effects and uncertainty explicitly we extend Lucas's (2000) benchmark welfare cost analysis in two additional ways. First, as in Obstfeld (1994a), Tallarini (2000), and Kenc (2004), we adopt a more general specification of preferences. We use a functional form of preferences that allows one to separate the roles played by agents' attitudes towards risk and intertemporal substitution.[2] Second, by using a general equilibrium model, we capture growth and welfare effects of variances. Moreover, we allow volatility to affect portfolio balance which may have growth and welfare effects. This is a direct result of mean-variance equilibrium feature of our model. Using this framework, we find a substantial welfare gain in the order of 21 percent of initial capital associated with bringing down the

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[1]See Rogers (2001) for calculations of welfare effects attributable to shifts in portfolio balances.

[2]Distinguishing between the two preferences parameters for risk aversion and intertemporal substitution has been found to lead to improvements on conventional measures of welfare cost of business cycle, see Obstfeld (1994a).

inflation rate from a baseline of about 3.4 percent to the socially optimum level. As for monetary volatility, the representative household would be willing to forego a trivial fraction of its initial capital (about 0.08 percent) to avoid monetary variability.

Section 2 of the paper outlines our development of existing continuous time stochastic endogenous growth models. In Section 3 we provide numerical estimates for the welfare cost of inflation; and in Section 4 we conclude the paper.

2 The Model

We use a money-in-the-utility function representative agent model and a Rebelo (1991) 'AK' production function which leads to endogenous growth. The stochastic nature of the model is characterized by two exogenous shocks. One of these is a productivity shock; the other is a monetary growth shock (a form of policy shock). For simplicity we choose a closed economy in which we capture the full and well-defined quantitative effects of monetary volatility without having exogenous effects coming from the rest of the world. The behavioral nature of the model is described by the utility maximizing and portfolio optimizing behavior of a representative household. The household's period utility function depends on both consumption and real cash balances. The introduction of money in the utility function is widely used [see for example Stulz (1986), Turnovsky (1993), and Basak and Gallmeyer (1999)]. This is not only for its tractability but also for its ability to capture money's role as a store of value and a medium of exchange [see Feenstra (1986) and Danthine, Donaldson, and Smith (1987)] [3]. The paper deals with only the steady-state stochastic equilibrium which is separated into deterministic and stochastic components.

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[3]Adopting an alternative specification for money such as a cash-in-advance constraint entails computational difficulties unless one restricts the sources of uncertainty in the model; see Rebelo and Xie (1999).

At each point in time the representative household chooses its consumption $C(t)$ and allocates its portfolio of wealth, $W(t)$, across three assets: money M, bonds B, and capital K. To facilitate a neat solution we assume that the net supply of bonds in equilibrium is zero. The only source of income for the representative household is the capital income received from holding these assets.

Geometric Brownian motion processes govern the two exogenous stochastic shocks. As a result, endogenous variables such as prices and returns evolve according to

$$dx/x = (\text{drift term})dt + (\text{diffusion parameter})dZ \tag{1}$$

where x is an endogenous variable such as $P = $ prices. Thus, for example, $\pi\, dt$ is the expected mean rate of change of P. dZ is the increment of the Brownian motion Z which is an i.i.d., standard normal process.

Incorporating money into the model gives rise to a separation of real returns from nominal returns. Return to the productive asset, capital, will be described below but returns to other assets can be described in terms of the interest rates they pay. Bonds pay nominal rates of interest i. Applying stochastic calculus, we obtain the real rates of return to the holdings of money and bonds as follows:

$$dR_M = r_M dt - dZ_p \qquad\qquad r_M = -\pi + \sigma_p^2 \tag{2a}$$

$$dR_B = r_B dt - dZ_p \qquad\qquad r_B = i - \pi + \sigma_p^2 \tag{2b}$$

The real rate of return to equity holders is calculated from the flow of new output dY per capital K. We assume that output is produced from capital by means of the stochastic constant returns to scale technology; and the economy-wide capital stock is assumed to have a positive external effect on the individual factor capital. We therefore write the aggregate production function as an AK function of the kind discussed by Rebelo (1991) with a stochastic linear coefficient

$$dY(t) = A[dt + dy(t)]K(t) \tag{3}$$

where A is the marginal physical product of capital and dy a productivity shock. Technically, dy represents increments to a Brownian motion with zero drift and variance $\sigma_y^2 dt$. Thus the return to capital before and after separating its deterministic and stochastic terms is respectively

$$dR_K = Adt + Ady \qquad\qquad r_K = Adt \qquad du_K = Ady. \qquad (4)$$

2.1 Household Optimization

At each point in time t a representative household holds three assets. The household's total wealth W consists of money M, bonds B, and equities K. Hence the wealth W constraint, in real terms, faced by the individual is

$$W = \frac{M}{P} + \frac{B}{P} + K$$

The government distributes the proceeds of seignorage revenue as transfers to households. The real value of these transfers is random because the government follows a stochastic monetary rule. We assume that transfer income dT is non-tradable[4] and in real terms remains proportional to wealth, as follows:

$$dT = \tau W dt + \sigma_v W dZ_v. \qquad (5)$$

The representative household constructs an optimal portfolio of its total wealth subject to the adding up condition for portfolio shares:

$$1 = n_M + n_B + n_K \qquad (6)$$

where $n_M = (M/P)/W$, $n_B = (B/P)/W$ and $n_K = K/W$. Consumers are assumed to purchase output over the instant dt at the nonstochastic rate $C(t)dt$ using the capital income generated from holding assets.

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[4]See Stulz (1986) for a model in which transfers are assumed to be tradable as well as non-tradable.

The representative agent's intertemporal utility is given by:

$$U(t) = e^{-\delta t}[\tilde{u}(t)^\alpha + e^{-\delta dt}V(t+dt)^\alpha]^{1/\alpha} \qquad (7)$$

$$V(t+dt) = [\mathcal{E}_t U(t+dt)^{1-\gamma}]^{1/(1-\gamma)}$$

$$\tilde{u}(t) = C(t)^\theta (M(t)/P(t))^{1-\theta}$$

where $\mathcal{E}$ is the expectation operator, δ is the time preference rate, $1/(1-\alpha)$ is the usual intertemporal substitution elasticity parameter and γ is the degree of relative risk aversion. $\tilde{u}$ represents the instantaneous utility function which is defined over consumption C and real money balances M/P. θ is the share of consumption in the instantaneous utility. This structure implies that utility satisfies intertemporal consistency of preferences and removes the restriction that $\alpha + \gamma = 1$.[5] The wealth accumulation of the representative household is given by:

$$\frac{dW}{W} = \psi dt + \sigma_w dZ_w \qquad (8a)$$

$$\psi = n_M r_M + n_B r_B + n_K r_K - \tau - C(t)/W(t) \qquad (8b)$$

$$\sigma_w dZ_w = -n_M \sigma_p dZ_p - n_B \sigma_p dZ_p + n_K A\sigma_y dZ_y - \sigma_v dZ_v \qquad (8c)$$

The solution strategy[6] works through the agent's value function, $\exp\{-\delta t\}V(W)$ which is the expected present discounted value of the agent's utility. The household

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[5] The utility function employed here disentangles risk aversion from intertemporal substitution as proposed by Epstein and Zin (1989) and Weil (1990). There are two reasons for choosing a utility function with this property: (i) it has been shown that dynamic welfare comparisons that conflate risk aversion and intertemporal substitutability can be misleading, Obstfeld (1994a); and (ii) one would like to answer questions about how preference parameters influence the numerical estimates of welfare costs. There are many ordinally equivalent representations of recursive representations that yield the same consumption and portfolio rules (see Duffie and Epstein (1992)): the particular formulation used in this paper was chosen by reference to a criterion of requiring tractability of the model (and differs slightly from that used in Grinols (1996)). Our formulation resembles that of Obstfeld (1994b).

[6] See e.g. Merton (1992) for a discussion of the methods. Details of the specific solution to various

maximizes utility by choosing the optimal full (composite) consumption-wealth ratio and the optimal portfolio shares of assets, taking the rates of return on assets, and the relevant variances and covariances as given[7].

The solution of consumption is then

$$\frac{C}{W} = \frac{\theta}{1 - \theta\alpha}\left[\delta - \alpha\left(r_Q - \tfrac{1}{2}\gamma\sigma_w^2\right)\right], \tag{9a}$$

where $r_Q = n_M r_M + n_B r_B + n_K r_K - \tau$. We define $r_Q - \tfrac{1}{2}\gamma\sigma_w^2$ as the risk adjusted rate of return which is denoted by r_A. Expression (9a) reveals that the optimal consumption and saving decision depends on both the intertemporal elasticity of substitution and the coefficient of risk aversion. The optimal portfolio shares are given by

$$n_M = \left[\frac{1-\theta}{\theta}\right]\left[\frac{C/W}{i}\right], \tag{9b}$$

$$(r_K - r_B)dt = \gamma cov(\sigma_w dZ_w, A\sigma_y dZ_y + \sigma_p dZ_p). \tag{9c}$$

2.2 Government Policy

The government engages in two activities, (i) printing money and (ii) distributing the receipts as transfers. The government pursues the following monetary policy rule:

$$dM/M = \mu dt + \sigma_m dZ_m \tag{10}$$

where μ is the mean monetary growth rate, σ_m is the volatility of the monetary growth rate, normally distributed and independent over time with zero mean and variance

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problems can be found at length in Turnovsky (1995). Our solution is available in the appendix to this paper posted on the web. The solution for an open economy case of the model used here can be found in Evans and Kenc (2003).

[7]However, the general equilibrium conditions, i.e. market-clearing conditions, of the model will determine these rates of return, variances and covariances.

$\sigma_m^2 dt$. dZ_m is the increment of Brownian motion which is allowed to be correlated with dZ_y, the random component of production.[8]

2.3 Macroeconomic Equilibrium and Determination of Variables

Goods market equilibrium in our model leads to the following expression for the rate of growth of the capital stock:

$$\frac{dK}{K} = \left(A - \frac{1}{n_K} \frac{C}{W} \right) dt + A\sigma_y dZ_y. \tag{11}$$

The derivation of the remaining equilibrium takes place in two stages. The first stage involves the determination of stochastic components. The second stage involves substitution of these solutions into the deterministic components of the equilibrium.

2.3.1 Calculation of the Stochastic Components

The stochastic adjustments in the economy derive from (1) the stochastic adjustment in the price level; (2) the stochastic component of wealth; and (3) the stochastic adjustment in transfer payments.

One can obtain stochastic terms from the above relationships and then use them to calculate the endogenous variances and covariances that appear in the first order optimality conditions and elsewhere.

$$\sigma_w^2 = A^2 \sigma_y^2 \tag{12a}$$

$$\sigma_p^2 = \sigma_m^2 + A^2 \sigma_y^2 \tag{12b}$$

$$cov(\sigma_w dZ_w, A\sigma_y dZ_y + \sigma_p dZ_p) = [A^2 \sigma_y^2 + \sigma_{wm}]dt \tag{12c}$$

$$cov(\sigma_w dZ_w, dp) = [-A^2 \sigma_y^2 + \sigma_{wm}]dt \tag{12d}$$

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[8]This correlation may have non-trivial growth and welfare effects [see Stulz (1986)]. Similar models such as Turnovsky (2000) largely abstract from such features.

2.3.2 Calculation of the Deterministic Components

Substituting for r_B and r_K from (2b) and (4) together with expressions (12a)–(12d), the first order condition for the optimal portfolio share n_K, the optimal portfolio share is given by:

$$(r_K - r_B)dt = \gamma\,cov(dw, Ady + dp), \tag{13}$$

which can be rewritten as:

$$A - i + \pi - \sigma_p^2 = \gamma[A^2\sigma_y^2 + \sigma_{wm}] \tag{14}$$

With portfolio shares remaining constant over time, all real components of wealth must grow at the same stochastic rate. That is

$$\frac{d(M/P)}{M/P} = \frac{d(B/P)}{B/P} = \frac{dK}{K} = \frac{dW}{W} = \psi dt + dw \tag{15}$$

Taking expectations of the accumulation equation (11), using (15), the real rate of growth is given by the expression

$$\psi = A - \frac{1}{n_K}\frac{C}{W} \tag{16a}$$

Using the rules of stochastic calculus, the left hand side of (15) (after substituting, simplifying and equating the resulting expression to (15)) yields an expression for the inflation rate

$$\pi = \mu - \left(A - \frac{1}{n_K}\frac{C}{W}\right) + \sigma_y^2 - \sigma_{ym} \tag{16b}$$

Substituting for (12a) in the solution for consumption (9a) yields an expression for C/W

$$\frac{C}{W} = \frac{\theta}{1 - \theta\alpha}\left[\delta - \alpha\left(r_Q - \tfrac{1}{2}\gamma A^2\sigma_y^2\right)\right]. \tag{16c}$$

9

where $r_Q = n_M r_M + n_B r_B + n_K r_K - \tau$, a function of the mean growth rate. As before, we define $r_Q - \frac{1}{2}\gamma\sigma_w^2$ as the risk adjusted rate of return which is denoted by r_A; and expression (16c) again reveals that the optimal consumption and saving decision depends on both the intertemporal elasticity of substitution and the coefficient of risk aversion.

The optimal portfolio share of money is given by

$$n_M = \left[\frac{1-\theta}{\theta}\right]\left[\frac{C/W}{i}\right].$$

(16d)

From the portfolio shares adding up condition (6) we obtain an expression for n_K

$$n_K = \frac{[r_K - r_B] - \gamma(\sigma_{yv} + \sigma_{pv} - \sigma_{yp} - \sigma_p^2)}{\gamma(\sigma_y^2 + \sigma_p^2 + 2\sigma_{yp})}$$

(16e)

From (14) we obtain i

$$i = A + \pi - \sigma_p^2 - \gamma[A^2\sigma_y^2 - \sigma_{wm}].$$

(17)

2.4 Modelling Monetary Policy Uncertainty

The assumption that the drift and diffusion terms of all stochastic processes are constant implies that investors' probability beliefs converge instantaneously to true probabilities. This assumption was relaxed by Stulz (1986) building on Williams (1977). He develops a model in which optimizing households are uncertain about the distribution of monetary growth and learn about it over time. The imperfect information case implies that the households' predictive distribution over money growth has a higher variance than the variance of money growth. Households can compute their predictive distribution available to them. This predictive distribution can be derived explicitly for the case in which households have a diffuse prior before sampling and their only relevant information is the time series of changes in the money supply. In this case, the predictive distribution is normal with mean

$$\mu^e(t) = \tfrac{1}{2}\sigma_X^2 + \frac{1}{t}\ln\frac{M(t)}{M(0)}$$

(18)

per unit of time and variance $((t+1)/t)\sigma_X^2 = \omega^2 \sigma_X^2$ per unit of time. t is the time elapsed since the monetary policy was introduced. For households to be uncertain about the mean growth rate of the monetary stock, it is required that $t < \infty$. In the following, ω is used to measure the degree of monetary policy uncertainty. When $\omega = 1$, there is no monetary policy uncertainty and households know the true dynamics of the money stock. Finally, μ^e follows

$$d\mu^e(t) = \frac{1}{t}\frac{dM}{M} - \frac{1}{t}\mu^e(t)dt \tag{19}$$

In this model, by construction, there is no uncertainty about the instantaneous variance of the growth rate of the money supply because households sample continuously.

2.5   Logarithmic Utility

In this section, we derive the stochastic process generating the equilibrium inflation rate under a restricted set of preference parameters. Even though a full version of the model is used, the preferences are restricted such that the representative agent has a logarithmic utility function. This enables us to obtain explicit solutions to the equilibrium portfolio shares and the inflation rate and assess the welfare effects of monetary policy[9].

In particular, with logarithmic utility, the equilibrium portfolio shares of real money balances and equity along with the equilibrium expected inflation rate reduce to:

$$n_M = \frac{(1-\theta)}{i}\delta \tag{20}$$

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[9]In this case the intertemporal utility function is given by

$$U_t = \int_t^\infty e^{-\delta s}\Big[\theta \ln C_s + (1-\theta)\ln(M_s/P_s)]\Big]ds.$$

11

$$n_K = \frac{A + \pi - i - n_M \sigma_m^2 + \sigma_{ym} - \sigma_y^2}{\sigma_m^2} \tag{21}$$

$$\pi = \mu - (A - \frac{\delta\theta}{1 - n_M}) + \sigma_y^2 - \sigma_{ym} \tag{22}$$

Given the portfolio add-up constraint $n_M + n_K = 1$, equations (20) through (21) imply a quadratic expression for the nominal interest rate,

$$i^2 - (\delta + [\mu - \sigma_m^2])i + (1 - \theta)\delta[\mu - \sigma_m^2] = 0 \tag{23}$$

with the following roots:

$$i = \tfrac{1}{2}\{[\delta + (\mu - \sigma_m^2)] \pm \tfrac{1}{2}\sqrt{[\delta + (\mu - \sigma_m^2)]^2 - 4(1 - \theta)\delta[\mu - \sigma_m^2]} \tag{24}$$

The second order conditions for welfare maximization and positive portfolio shares in equilibrium, $0 < n_M < 1$ and $0 < n_K < 1$, imply that the larger of the two roots is the equilibrium solution for the nominal interest rate.

One can use (21) through (22) and (24) to derive the effects of changes in the distribution of the money growth on the equilibrium expected inflation rate:

$$\frac{d\pi}{d\mu} = 1 + \frac{\partial\pi}{\partial n_M}\frac{\partial n_M}{\partial i}\frac{\partial i}{\partial \mu} \tag{25}$$

$$\frac{d\pi}{d\sigma_m^2} = \frac{\partial\pi}{\partial n_M}\frac{\partial n_M}{\partial i}\frac{\partial i}{\partial \sigma_m^2}. \tag{26}$$

Evidently, the mean growth rate of money has a direct effect on expected inflation and an indirect effect through portfolio allocation and the equilibrium growth rate. The variance of money growth influences expected inflation insofar as it affects the equilibrium nominal interest rate, portfolio balance, and the equilibrium growth rate.

These respective effects can be calculated as follows:

$$\frac{\partial\pi}{\partial n_M} = \frac{\theta\delta}{(1 - n_M)^2} > 0; \tag{27}$$

$$\frac{\partial n_M}{\partial i} = -\frac{(1-\theta)\delta}{i^2} < 0; \tag{28}$$

$$\frac{\partial i}{\partial \mu} = \frac{1}{2} + \frac{\delta + (\mu - \sigma_m^2) - 2(1-\theta)\delta}{\sqrt{[\delta + (\mu - \sigma_m^2)]^2 - 4(1-\theta)\delta[\mu - \sigma_m^2]}} > 0; \tag{29}$$

$$\frac{\partial i}{\partial \sigma_m^2} = -\frac{1}{2} - \frac{\delta + (\mu - \sigma_m^2) - 2(1-\theta)\delta}{\sqrt{[\delta + (\mu - \sigma_m^2)]^2 - 4(1-\theta)\delta[\mu - \sigma_m^2]}} < 0. \tag{30}$$

As in Gertler and Grinols (1982), the signs in (29) and (30) follow from portfolio balance and are derived under reasonable parameter values. An increase in the mean money growth rate μ raises expected inflation directly since in a balanced growth path, the ratio of real money balances to equity has to be maintained. However, as shown in (27) through (29), the increase in money growth raises the nominal interest rate in order to maintain portfolio balance, and consequently reduces money holdings and increases the demand for capital thereby raising the equilibrium growth rate. The latter somewhat reduces expected inflation. An increase in the variance of money growth raises the variance of the inflation rate which makes both money and bonds more attractive relative to capital. As such, the nominal interest rate falls, money holdings rise, equity holdings and investment fall, and equilibrium growth rate falls; consequently expected inflation rises.

The effect of monetary policy on representative individual's welfare can be calculated explicitly in this case. The initial wealth stock is given by

$$W_0 = \frac{iK_0}{i - \delta(1-\theta)}. \tag{31}$$

The welfare criterion can be written as

$$J(W) = \frac{1}{\delta^2}\left(\psi - \frac{1}{2}\sigma_W^2\right) + \frac{1-\theta}{\delta}\ln n_M + \frac{\theta}{\delta}\ln\theta\delta + \frac{1}{\delta}\ln W. \tag{32}$$

Substituting initial wealth and other optimized values into the welfare criterion, and differentiating (32) with respect to the nominal interest rate

$$\frac{dJ_0(i, K_0)}{di} = \frac{1}{\delta}\left(\frac{1}{i} - \frac{1}{i - \delta(1 - \theta)}\right) - \frac{1 - \theta}{\delta i} + \frac{\theta(1 - \theta)}{[i - \delta(1 - \theta)]^2}. \tag{33}$$

As emphasized by Turnovsky (1993), the first effect in this expression is an initial price jump effect. An increase in the nominal interest rate causes an increase in the initial price level to restore equilibrium in stock terms. This reduces initial wealth and is welfare deteriorating. The second effect is a money demand effect where the increase in the interest rate reduces equilibrium real money balances and welfare. The third effect is a portfolio allocation effect where a higher interest rate reduces mean consumption growth and increases saving, capital accumulation, and growth. Which effect dominates cannot be determined unambiguously. However, rewriting equation (33), we obtain

$$\frac{dJ_0(i, K_0)}{di} = \frac{-i^2 + \delta(1 - \theta)i + \delta^2\theta(1 - \theta)}{\delta i[i - \delta(1 - \theta)]^2}. \tag{34}$$

It is evident that the effect of the nominal interest rate on welfare depends on the sign of the nominator in (34). The optimum interest rate is given by the root of the quadratic equation

$$-i^2 + \delta(1 - \theta)i + \delta^2\theta(1 - \theta) = 0 \tag{35}$$

and the welfare maximizing interest rate is given by the positive root of the same equation:

$$i* = \left(\tfrac{1}{2}(1 - \theta) + \tfrac{1}{2}\sqrt{1 + 2\theta - 3\theta^2}\right)\delta.$$

It is clear that the optimum interest rate is independent of other sources of risk and government policy. As emphasized by Turnovsky (1993), logarithmic preferences insulate the optimum interest rate from all sources of risk when, in a more general preference setting, the consumption wealth ratio, and hence the nominal interest rate, would depend on wealth risk.

14

3 Welfare Calculations

This section considers the welfare effects of (i) a monetary policy aimed at cutting down the inflation to the optimal rate; (ii) the monetary policy in (i) under policy uncertainty, and (iii) the elimination of monetary volatility. To measure these welfare effects we focus on the welfare of the representative agent and first evaluate the expected lifetime utility associated with the optimal consumption path:

$$\mathcal{E}_0(U) = \Lambda \frac{W(0)^{1-\gamma}}{1-\gamma} \tag{36}$$

where

$$\Lambda = \left\{ \frac{\theta}{C/W} \left[(C/W)^\theta n_M^{1-\theta} \right]^\alpha \right\}^{\frac{1-\gamma}{\alpha}}$$

Substituting and simplifying:

$$\Lambda = (C/W)^{-\frac{(1-\alpha)(1-\gamma)}{\alpha}} \theta^{\frac{(1-\gamma)}{\alpha}} \left[\frac{1-\theta}{\theta} \right]^{(1-\theta)(1-\gamma)} i^{-(1-\theta)(1-\gamma)} \tag{37}$$

Then, following Barlevy (2004), Pallage and Robe (2003), and Epaulard and Pommeret (2003) we utilize a definition of the welfare cost as follows.

Definition: *The welfare cost is defined as the percentage of capital the representative agent is ready to give up in period zero to be as well off in a particular world, $(\Psi(\Omega_j), \Omega_j)$, as she is in the baseline case, $(\Psi(\tilde{\Omega}), \tilde{\Omega})$, (i.e., it is a 'compensating variation measure')*[10]. *Ω is the usual variance-covariance matrix and Ψ summarizes the expected mean growth rate which in this model is affected by changes in volatility unlike many of the earlier models used in the literature.*

Thus, denoting the cost by κ

$$\mathcal{E}_0[U(K(0), (\Psi(\tilde{\Omega}), \tilde{\Omega})] = \mathcal{E}_0[U((1-\kappa)K(0), \Psi(\Omega_j), \Omega_j)] \tag{38}$$

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[10]Obstfeld (1994b) obtains an 'equivalent variation measure'.

Using (36), the welfare cost of policy may be written:

$$\kappa = 1 - \left(\frac{\Lambda(\Psi(\tilde{\Omega}), \tilde{\Omega})}{\Lambda(\Psi(\Omega_j), \Omega_j)} \right)^{1/(1-\gamma)} \left[\frac{n_K(\Omega_j)}{n_K(\tilde{\Omega})} \right] \tag{39}$$

Following Barlevy (2004) and Pallage and Robe (2003) we also compute welfare measure in terms of initial consumption in order to make our welfare measure comparable with those of Lucas (1987). Under the compensating variation measure the corresponding cut in consumption, cic, the agent experiences when reaching a particular level of volatility Ω_j may expressed as:

$$\begin{aligned}
(1 - cic)\tfrac{C}{W}(\Omega_j) &= \tfrac{C}{W}\left(\tilde{\Omega}\right)(1 - \kappa) \\
cic &= 1 - \frac{\tfrac{C}{W}(\tilde{\Omega})}{\tfrac{C}{W}(\Omega_j)}(1 - \kappa)
\end{aligned} \tag{40}$$

Equation (39) or (40) can be used to quantify the effects of policy changes on economic welfare. In this model, the government's policy parameter relates to monetary growth; and the exogenous stochastic processes include monetary growth (dx) and productivity (dy). However, while equation (39) or (40) provides the basis for quantification of the welfare effects, the generation of numerical estimates requires the specification of a number of baseline parameters and variables. Table 1 sets out the values used in the numerical exercises carried out here.

[Table 1 approximately here.]

No particular claim is made for the precision of these numerical values; rather, the intention is to utilize plausible values. The values used are comparable with those used in similar exercises such as Dotsey and Ireland (1996), Lucas (2000), Turnovsky (2000), Barlevy (2004), Epaulard and Pommeret (2003), Evans and Kenc (2003) and Pallage and Robe (2003). Nevertheless, particular mention should be made of the values assigned to certain preference parameters, mean monetary growth rate, and variance-covariance terms.

- The mean monetary growth rate, $\mu = 0.06$, is chosen such that it yields an inflation rate of 3.4% which is comparable to those used in the literature.

- The consumption intensity, $\theta = 0.875$, is chosen such that it yields a steady-state consumption velocity (the ratio of C/W to n_M) close to unity - a value in line with that actually observed in the United States in the 1960-2000 period.

- Likewise, the standard deviation of output growth, $\sigma_y = 0.025$, the standard deviation of money supply growth, $\sigma_m = 0.041$, and the correlation coefficient between output growth and money growth, $\rho_{ym} = 0.084$, are chosen to closely match those observed in the United States in the 1960-2000 period.

- The risk aversion parameter is assigned the value 4, as in Obstfeld (1994b); this is the mid point of the range of conventional estimates (2 - 6) referred to in Obstfeld (1994a). However, we are mindful that some authors suggest that values of unity or values as high as 30 cannot be ruled out (see Epstein and Zin (1991); and Kandel and Stambough (1991), respectively).

- The intertemporal substitution elasticity is set to 0.35 which is in line with the literature; for example, Hall (1988), and Campbell and Mankiw (1989) suggest an intertemporal substitution elasticity of 0.10; Ogaki and Reinhart (1998) refer to the range 0.32 - 0.45. This is also comparable the value 0.5 used by Obstfeld (1994b) and is consistent with what Epstein and Zin (1991) describe as 'a reasonable inference'.

Later in this paper we calculate welfare costs for a range of values for the preference parameters and carry out a sensitivity analysis as to how these preference parameters influence the numerical estimates of welfare costs and the impact on the mean growth rate.

However, first we focus on calculating welfare costs for the different experiments using the values set out in Table 1. This involves identifying benchmark cases for each experiment. For example, the benchmark for the optimal inflation case assigns a zero welfare cost to the situation in which the inflation rate takes the baseline value. For each experiment, the bottom line impact on welfare is given in the final row of Table 2. The numerical value measures the welfare cost as a proportion of initial capital; and the sign (positive or negative) indicates whether welfare is damaged or improved, respectively. Thus for example, the representative agent would be willing to give up 21.16 per cent of their initial capital to live in a world with the optimal inflation.

3.1   Welfare effects of optimal inflation

In this section we calculate the welfare effects of alternative levels of inflation including an optimal level. These levels are obtained by adjusting the mean monetary growth rate μ. Table 2 reports selected statistics for the optimal inflation case. Of itself, this result is not surprising: it would generally be expected that the inflation damages welfare and thus that its elimination would enhance overall welfare. However, it can also be seen from Table 2 that the elimination of inflation has had a depressing effect on growth. What is interesting here is that the damaging effect on growth (and welfare) is being dominated by other influences on welfare from within the model. These other influences are two-fold - because the disinflationary policy impacts welfare through its effect on real money balances, initial wealth and growth. From Table 2, it can be seen that money balances have risen as has initial wealth[11] - both welfare enhancing; and the numerical solutions indicate that these dominate

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[11]Initial real wealth rises as a result of the initial price jump downward required to maintain portfolio balance in stock terms. The fall in the interest rate has generated an increase in initial real wealth (see Turnovsky (1993)).

the welfare deteriorating effect of lower growth. The intuition behind our result is as follows. The growth effect is brought about by the fact that the real rate of return on money rises under the optimal inflation rate and thereby leads to a portfolio shift from capital (the productive asset), whose real rate of return is constant, to money (nominal asset).

[**Table 2 approximately here.**]

Figure 1 shows how the welfare cost of inflation changes with the inflation rate (top panel). It also plots the interest and mean monetary growth rates corresponding to the welfare effects (middle and bottom panels, respectively). Figure 1 has the following notable feature. The shape of welfare costs broadly conforms to those in the literature [Braun (1994), Lucas (2000) and Cavalcanti and Villamil (2003)]. The welfare costs reflect an earlier well-known result with respect to optimum monetary policy and the optimum inflation rate - the Friedman rule.[12] In our quantitative experiments, the socially optimum inflation rate is -9.55 percent. However, this optimum inflation cannot be achieved by a zero interest rate; the optimum nominal interest rate is in the order of 1.23 percent. Here a zero interest rate would leave savings and the capital stock at suboptimal levels; consequently growth and future consumption stream would be lower.[13]

[**Figure 1 approximately here.**]

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[12]According to the Friedman rule, policies that generate a zero nominal interest rate will lead to optimal resource allocations. In a world without uncertainty, this involves price stability or a price deflation at a constant rate.

[13]Work on optimal policy has shown that the Friedman rule is robust to a variety of environments including those with distortionary taxes [Chari and Kehoe (1999)]. However, if there are imperfections in the fiscal regime as in many developing countries, the Friedman rule is not optimal [Cavalcanti and Villamil (2003)].

Another feature of Figure 1 is the asymmetric nature of the costs of inflation. As the inflation rate falls from its optimal level, the welfare cost of inflation increases sharply. However, when inflation is above its optimal rate, the welfare cost increases only gradually. The reason for the sharp increase in the welfare cost of inflation when inflation is below the optimal level is that such policy would drive the nominal interest rate to near zero levels (cf. Equation (16d)); as such, the representative household would demand all money and no capital. Clearly such a policy is costly in terms of growth. This asymmetry is not as sharp as that found in Braun (1994), Lucas (2000) and Cavalcanti and Villamil (2003). In these studies as the nominal interest rate approaches zero, the household demand for money is unbounded and the government revenue requirements needed to offset the revenue loss due to deflation becomes substantially large. Moreover, the lost government revenue must be made up by distortionary taxes. In our model however, the loss of government revenue from deflating merely reduces transfers to households; it does not lead to distortionary taxes.

Table 2 presents welfare effects with the first column giving the effect of reducing the inflation rate from a baseline of about 3.4 percent, to the optimum level. The optimum policy is to deflate at the rate 9.54 per year which can be achieved by contracting the money supply at 8.85 percent per year. As a result the nominal interest rate drops from a baseline rate of 14.18 percent to 1.22 percent per year. The reduction in interest rate induces portfolio adjustments and growth effects. the portfolio share of money (n_M) increases from 7.77 percent to 54.57 percent. The mean equilibrium growth rate drops from 2.63 percent to 0.74 percent[14] thereby reducing the consumption-wealth ratio ($\frac{C}{W}$) from 7.72 to 4.66 percent. The increase in n_M is welfare enhancing whereas the reduction in $\frac{C}{W}$ decreases welfare. Together with the welfare improving effect of the initial price jump, the overall welfare is positive.

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[14]This demonstrates that money superneutrality does not hold.

Consequently, the representative household would be willing to pay 21.16 percent of their initial capital to bring about the optimal policy.[15]Even though welfare gains from attaining an optimum deflation of nearly 10 percent are substantial, one should not take these results to be plausible for policy purposes. First, the model in this paper is a long run growth model and it may take a significant time to realize all gains. More importantly, the model has no distortions or market imperfections; its setup is a first best world. In a world with government expenditures and distortionary taxes, and with market imperfections in credit, insurance, input and output markets, the results are likely to be different. For example, in a second best world where governments cannot raise revenue as in Cavalcanti and Villamil (2003) or when inflation "greases the wheels of the labor market" due to nominal wage rigidity as in Card and Hyslop (1997), a higher inflation level can be an optimal policy. Therefore models with distortions and market imperfections are important in understanding economically feasible welfare gains from reducing inflation.

3.2   Elimination of monetary shocks

In this section, we search for optimal inflation in a world with no monetary volatility. Elimination of nominal shocks changes the risk-return characteristics of assets and produces some minor changes in welfare estimates. The process by which this comes about is as follows. It can be seen from middle column of Table 2 that elimination of monetary shocks reduces monetary growth risk, giving rise to a decline in the variance of the price level and a fall in the interest rate. The fall in the nominal interest rate increases the portfolio share of money, which in turn reduces somewhat the portfolio

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[15]While directly not comparable in terms of model specifications and measures, our estimates tend to be much higher than those high-end estimates found in the literature. Moving from roughly comparable baseline inflation rates to an optimal rate generates 4.7 percent in Den Haan (1990), 2.2 percent in Gillman (1993), 3.4 percent in Wu and Zhang (2000) in welfare costs of inflation.

share of the productive asset resulting in a small reduction in the mean growth rate. Even though it is trivial, this impact on portfolio allocation and the mean growth rate is in contrast to the models of Lucas (1987) and Obstfeld (1994a) where the mean growth rate is restricted to be unaffected by volatility.

In turning to the impact on welfare, it is evident that the saturation level of real money balances has increased, as has the monetary contraction rate, and the associated optimal deflation rate. In this case, the welfare gain from attaining optimal policy is in the order of 21.24 percent of initial capital. As expected, in a world with less frictions, the welfare gain of optimal policy is higher.

The intuition behind the result is as follows. With monetary assets in the portfolio now less risky, even risk averse households can shift towards the riskier, productive assets in their portfolio whilst still maintaining the overall risk level of the portfolio. The portfolio rate of return rises - an adjustment fully reflected in the risk-adjusted rate of return; and there is a balancing of the portfolio in favor of the (riskier) productive asset, promoting growth and welfare.

3.3 Welfare effects of inflation under monetary policy uncertainty

We now quantify the welfare estimates of optimal inflation under monetary policy uncertainty. This is modelled as an adaptive learning process with a time horizon of 2 years. It is assumed that the monetary authority follows the same policy rule, i.e., $\mu = -0.0885$. This raises the variances of monetary growth and the price level due to learning, implying a higher rate of return on money. The portfolio share of money (equity) increases (decreases) as compared to that without learning, leading to a portfolio adjustment and growth effects. The overall effect on welfare is smaller

under learning:[16] it is now down to 19.71 percent of initial capital from 21.16 percent as reported in the last column of Table 2.

3.4   Sensitivity analysis

The evidence from the experiments conducted here suggests some clear portfolio adjustment, growth, and welfare effects of optimal inflation in different environments including monetary volatility and learning. However, it has been shown elsewhere that it may be inappropriate to place too much weight on the particular portfolio adjustment, growth and welfare effects presented in Table 2: different parameter values capturing the two elements of household preferences could bring about significant differences in both the growth effects and in the intertemporal utility of the representative agent - our measure of welfare.

To explore this possibility we carry out a sensitivity analysis for the two experiments considered above. For each experiment, we calculate the portfolio adjustments, growth effects, and the welfare costs for a range of values of the two preference parameters - risk aversion and intertemporal substitution. However, values should be chosen in such way that they satisfy both feasibility ($\epsilon \leq 1$) and transversality ($\gamma \geq 1$) conditions as demonstrated in Smith (1996a). In Figure 2 (top left and bottom right) we see the results of this sensitivity analysis for calculations of the portfolio adjustments, growth, and welfare effects of the optimal inflation policy. The figure reveals that our previous results now need careful qualification.

Under reasonable parameter values the result was that there would be a depressing effect upon growth yet welfare would be substantially enhanced. It is clear from

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[16]We also relax the assumption that the monetary authority follows the same policy rule. When the monetary authority optimizes under learning, the welfare effect does not change much. However, the channels through which the welfare effects are obtained are different. The higher variances now reduce the optimal deflation rate and thereby induce a lower price jump effect.

Figure 2 that this result is not robust to the range of preference parameters considered here: indeed not only is the scale of the growth and welfare effects varying but so is the direction. Having noted this we would highlight the following point. Of the two preference parameters, it is clear that variation in the elasticity of substitution has the greater impact on the growth and welfare measures. This is consistent with the results found in Weil (1990) and Smith (1996b).

Finally, we explored an alternative specification of introducing money as a cash-in-advance constraint following Rebelo and Xie (1999). Even though obtaining an analytical solution comes at the expense of assuming $\sigma_y = \sigma_m$, the welfare gains of an optimal inflation policy remain substantial in the order of 12 percent of initial capital.

[**Figure 2 approximately here.**]

4   Conclusions

We examined the welfare costs of inflation in a stochastic general equilibrium balanced growth model in continuous time with mean-variance optimization and recursive utility. Among other things, our explicit account for portfolio adjustment effects, monetary variability and policy uncertainty yield substantial welfare gains as compared to moderate estimates of the existing literature. A monetary policy that brings down inflation from the baseline (3.4 percent) to the optimal rate (-9.54 percent) has a welfare gain in the magnitude of 21.16 percent of initial capital. As for monetary policy uncertainty, it magnifies inflation variability leading to portfolio adjustments and growth effects. The overall welfare effect of learning is found to be modest. In another experiment, the representative household would be willing to forego a trivial proportion of initial capital to avoid monetary variability.

However, our numerical results that are obtained under reasonable parameter

values are not robust across a range of the preference (risk aversion and intertemporal substitution) parameters. As with other quantitative experiments, the results are not meant for policy guidance; rather the estimates are only suggestive. Proper care should be taken to account for government expenditure and finance, and market imperfections such as credit, insurance, input and output markets. Another fruitful avenue would be the extension of the adaptive learning framework to a Bayesian learning.

Table 1: Baseline Parameters and Variables

Variable	Symbol	Value[a] %
Parameters		
Marginal product of capital	A	11.000
Risk aversion parameter	γ	4.000
Intertemporal substitution elasticity	$\epsilon = 1/(1-\alpha)$	0.350
Rate of time preference	δ	3.750
Consumption intensity	θ	0.875
Money growth rate	μ	6.000
Standard deviation of output	σ_y	0.025
Standard deviation of money supply	σ_m	0.041
Correlation coefficient ratio	ρ_{ym}	0.084
Variables		
Inflation rate	π	3.426
Standard deviation of price level	σ_p	0.046
Interest rate	i	14.182
Rate of return on money	r_M	-3.216
Rate of return on bonds	r_B	10.966
Rate of return on capital	r_K	11.000
Portfolio share of money	n_M	7.777
Portfolio share of equity	n_K	92.223
Rate of return on portfo lio	r_Q	10.349
Risk adjusted rate of return	$r_Q - \frac{1}{2}\gamma\sigma_w^2$	10.224
Consumption–wealth ratio	$\frac{C}{W}$	7.721
Mean equilibrium growth rate	ψ	2.628
Standard deviation of growth rate	σ_w	0.025

[a] All values are expressed as percentage expect for γ, ϵ, θ and variance-covariance terms.

Table 2: Welfare Effects[a]

Variable	Symbol	Optimal Inflation		
		With Monetary Volatility	Without Monetary Volatility	with Learning
Mean monetary growth rate	μ	-8.85	-9.03	-8.85
Inflation rate	π	-9.54	-9.71	-8.00
Standard deviation of price level	σ_p	4.58	2.50	6.37
Interest rate	i	1.22	1.22	2.54
Rate of return on money	r_M	9.75	9.78	8.41
Rate of return on bonds	r_B	10.97	11.00	10.95
Portfolio share of money	n_M	54.57	54.46	34.47
Portfolio share of equity	n_K	45.43	45.54	65.53
Rate of return on portfo lio	r_Q	5.40	5.42	7.78
Risk adjusted rate of return	$r_Q - \frac{1}{2}\gamma\sigma_w^2$	5.28	5.29	7.65
Consumption–wealth ratio	$\frac{C}{W}$	4.66	4.67	6.13
Mean equilibrium growth rate	ψ	0.74	0.75	1.65
Standard deviation of growth rate	σ_w	2.50	2.50	2.50
Compensating variation	κ	21.16	21.24	19.71
Corresponding cut in consumption	cic	13.16	13.24	15.75

[a] All values are expressed as percentage.
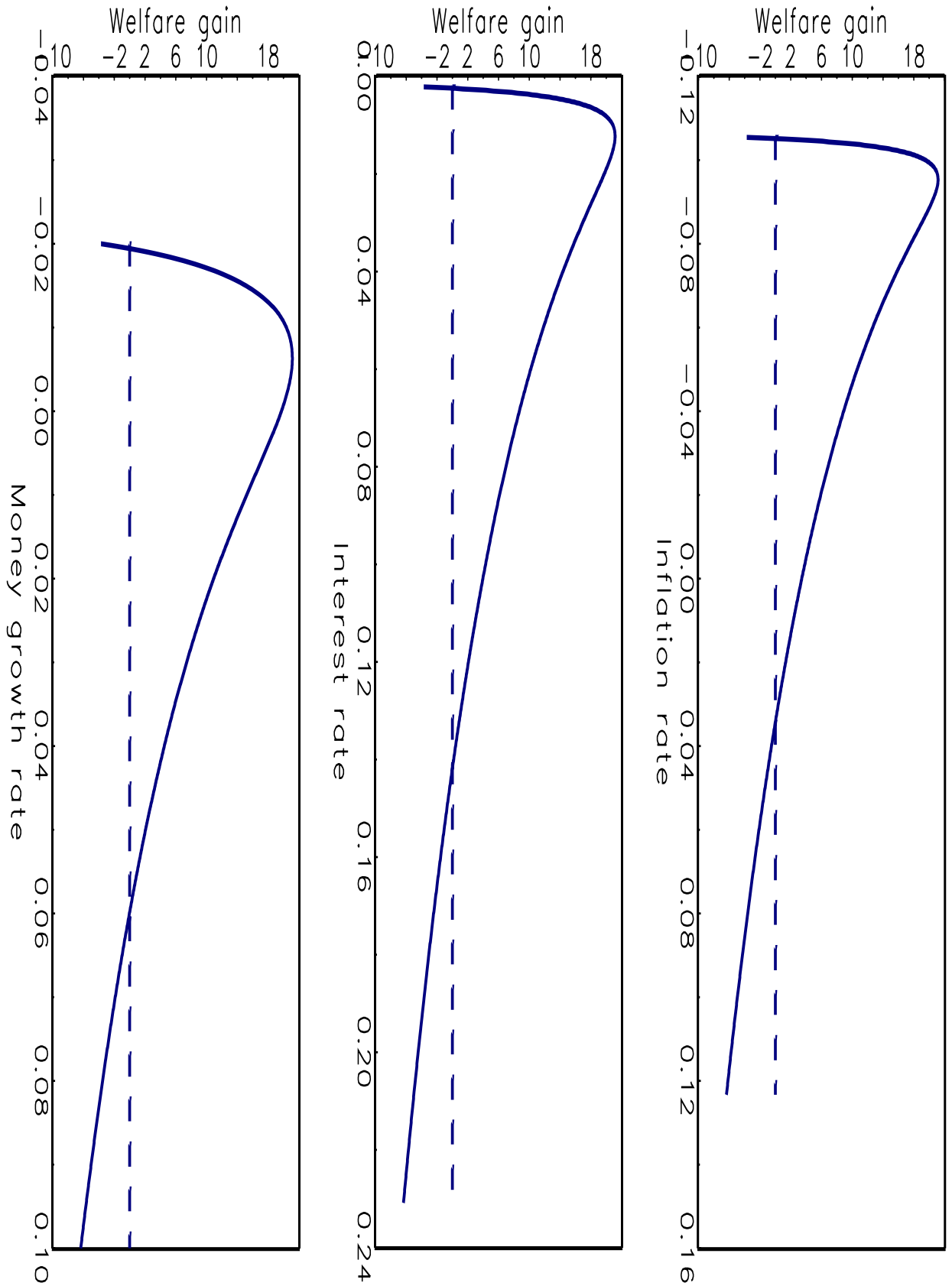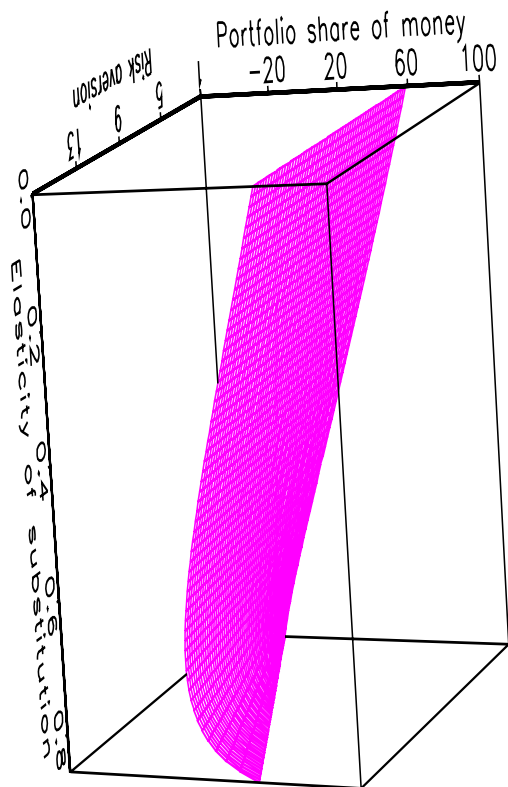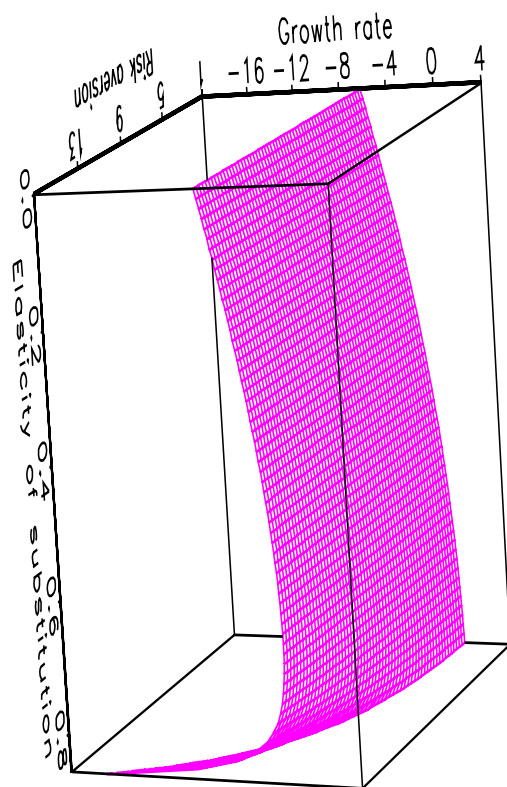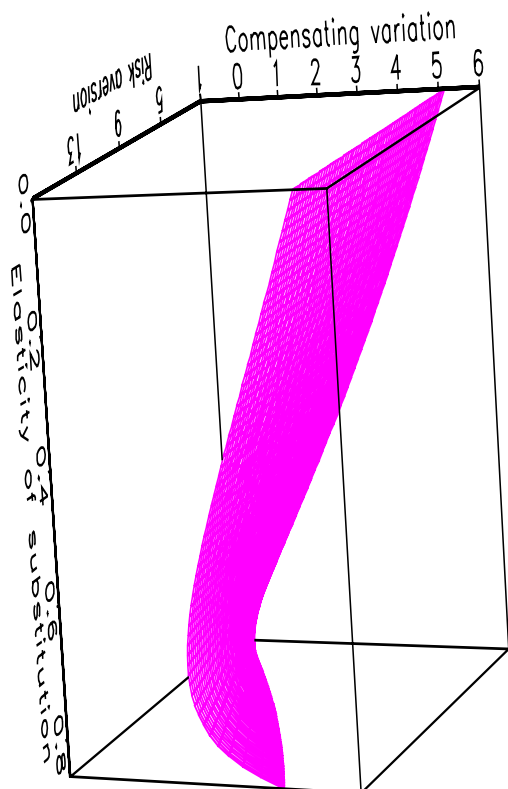
Figure 1: Welfare cost of inflation

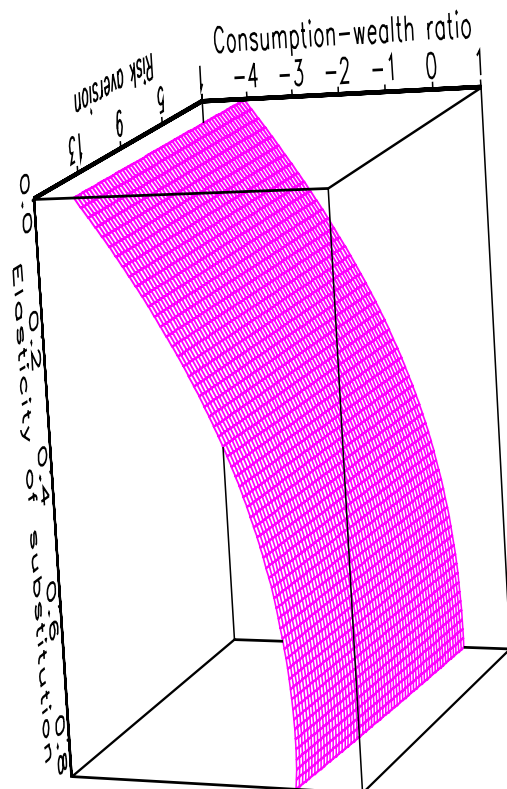Figure 2: Intertemporal substitution, risk aversion, and the welfare cost of inflation

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